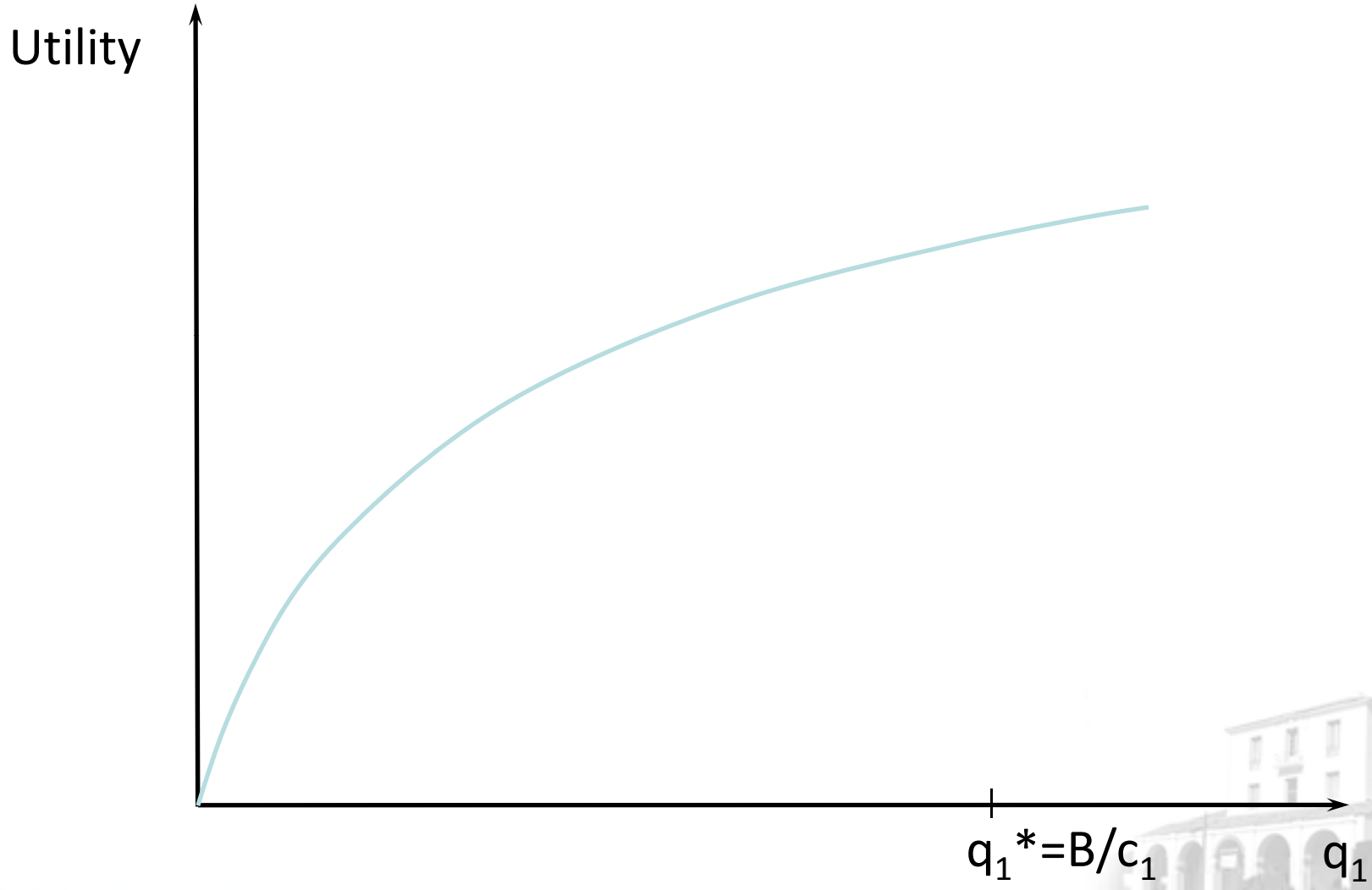
On a Quantitative Definition of Affordability

Charles J. LaCivita & Kent D. Wall
Naval Postgraduate School

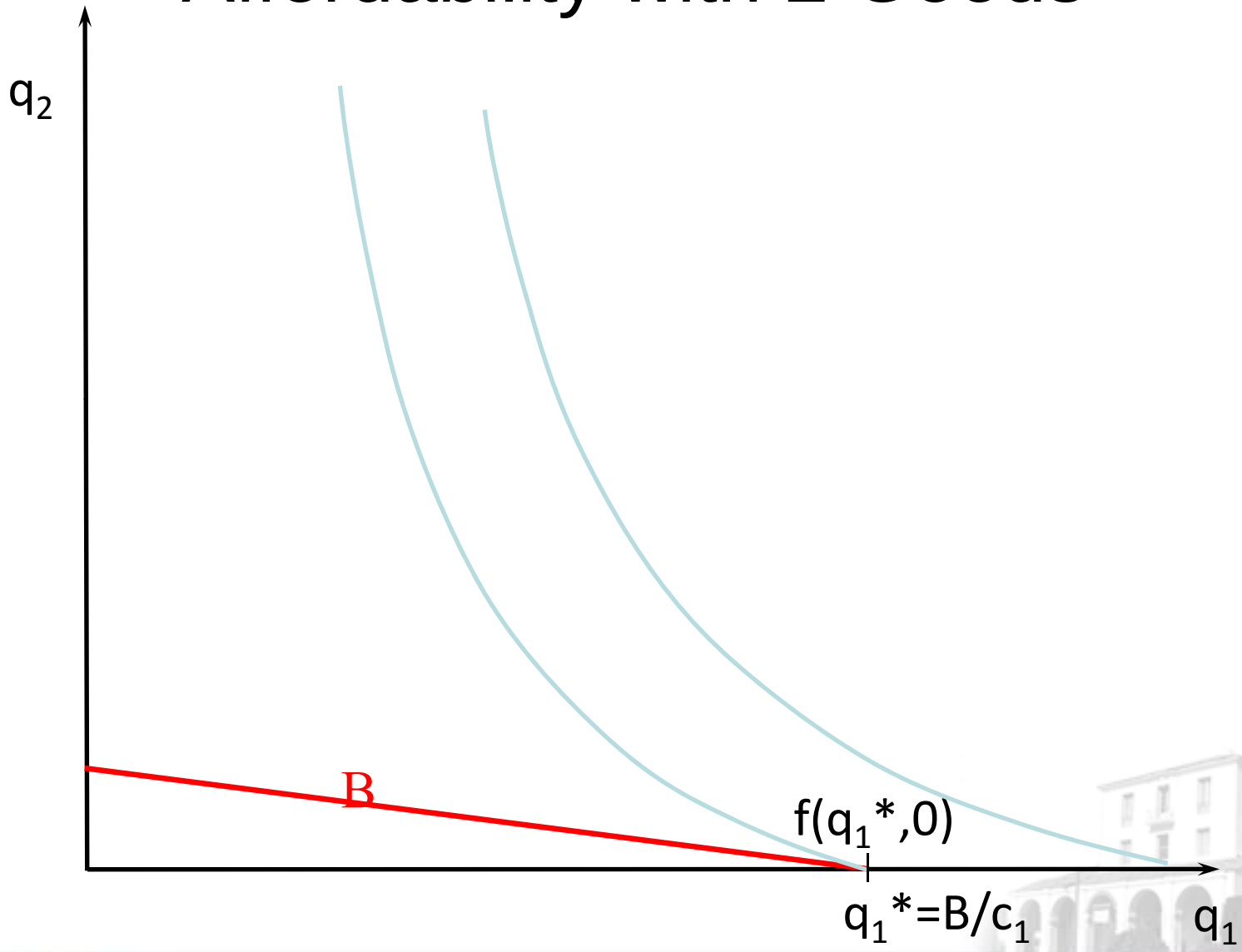
Definitions of Affordability

- No precise definition
- Definitions usually refer to:
  - Budget
  - Performance level
  - Life cycle costs

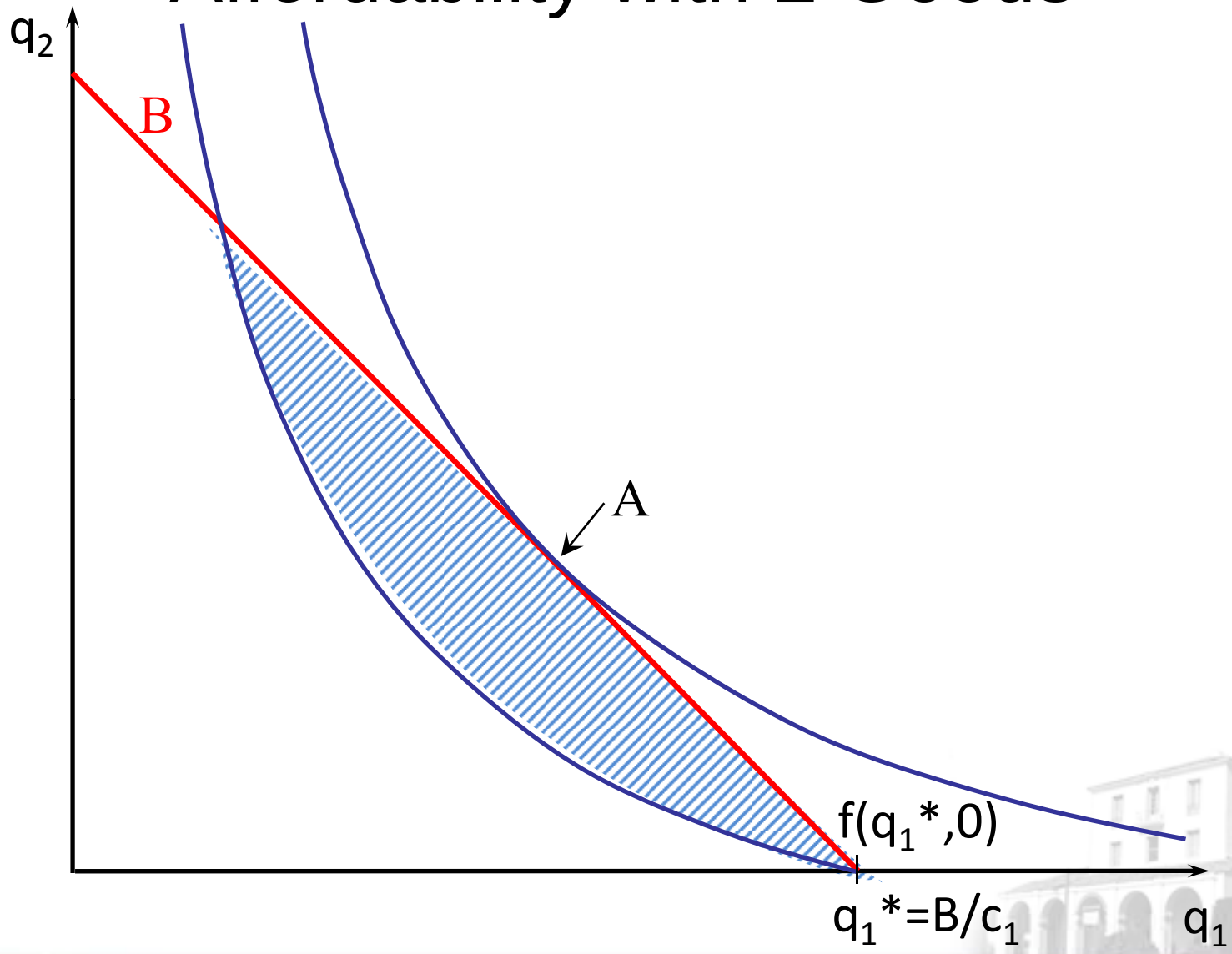
Affordability with 1 Good



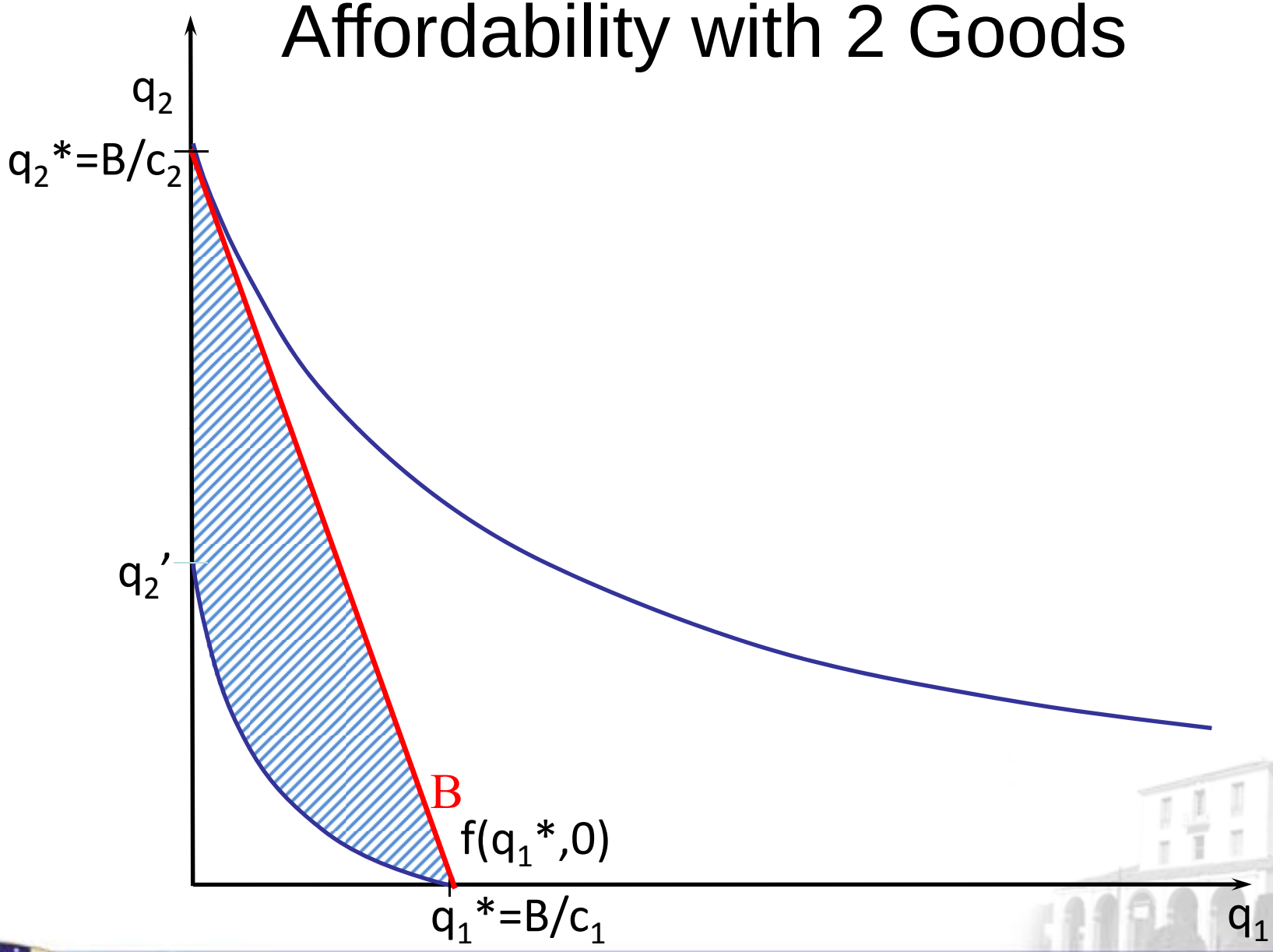
Affordability with 2 Goods



Affordability with 2 Goods



Affordability with 2 Goods



Index of Affordability

The ratio of the set of combinations of q_1 and q_2 that produce at least as much utility as q_1* to the set of all combinations of q_1 and q_2 that cost as much or less than q_1*. This ratio defines an affordability index α such that $0 \leq \alpha < 1$.

Affordability Example

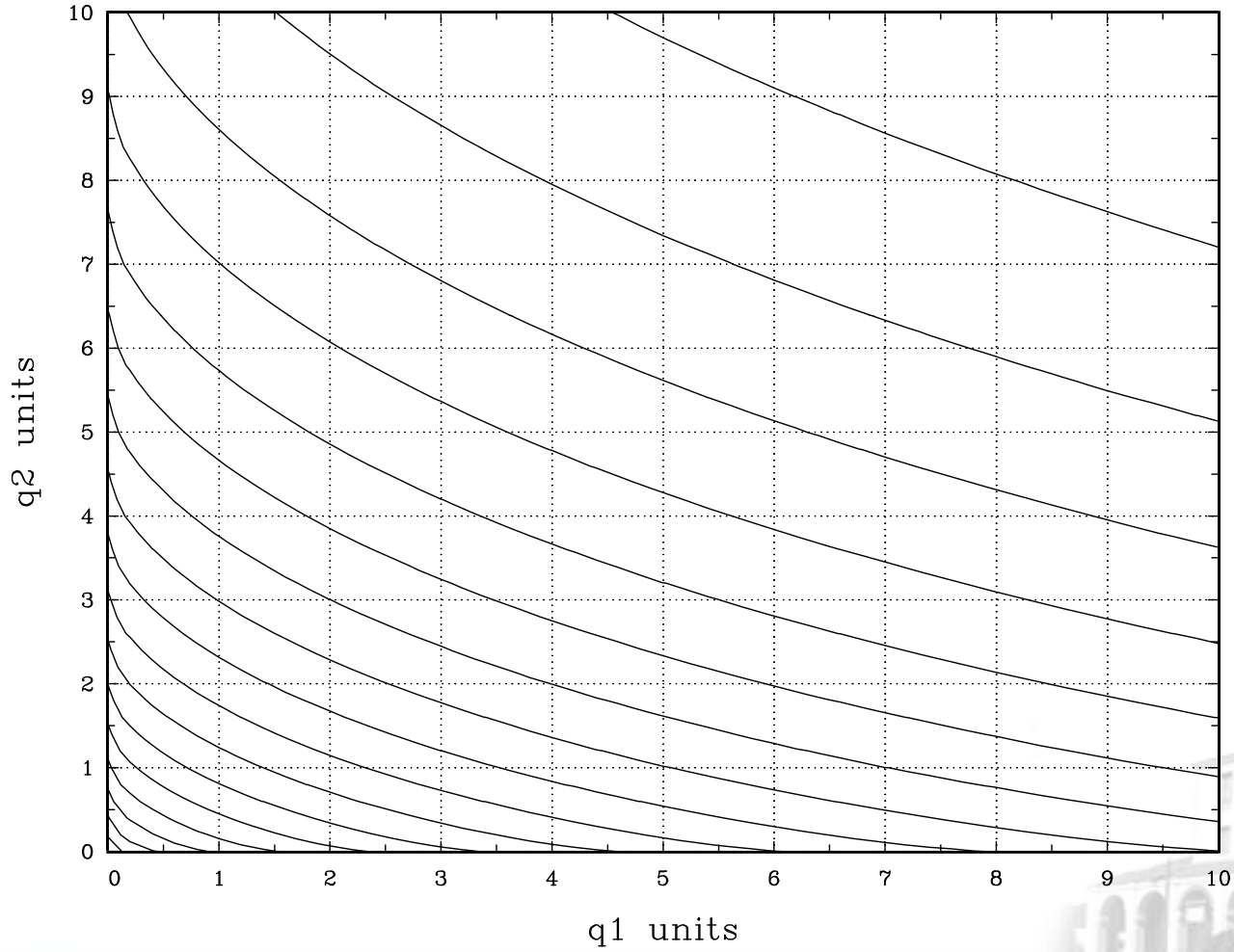
Individual effectiveness: $v_i(q_i) = 1 - e^{-g_i(q_i)}$

where $g_i(q_i) = a_i q_i^{b_i}$.

Joint effectiveness: $v(q_1, q_2) = 1 - e^{-f(q_1, q_2)}$

where $f(q_1, q_2) = g_1(q_1) + g_2(q_2) + d \cdot g_1(q_1) \cdot g_2(q_2)$.

$$a_1 = 0.16\ ,\ b_1 = 0.60\ ;\ \ a_2 = 0.21\ ,\ b_2 = 0.80\ ;\ \ d = 0.30$$

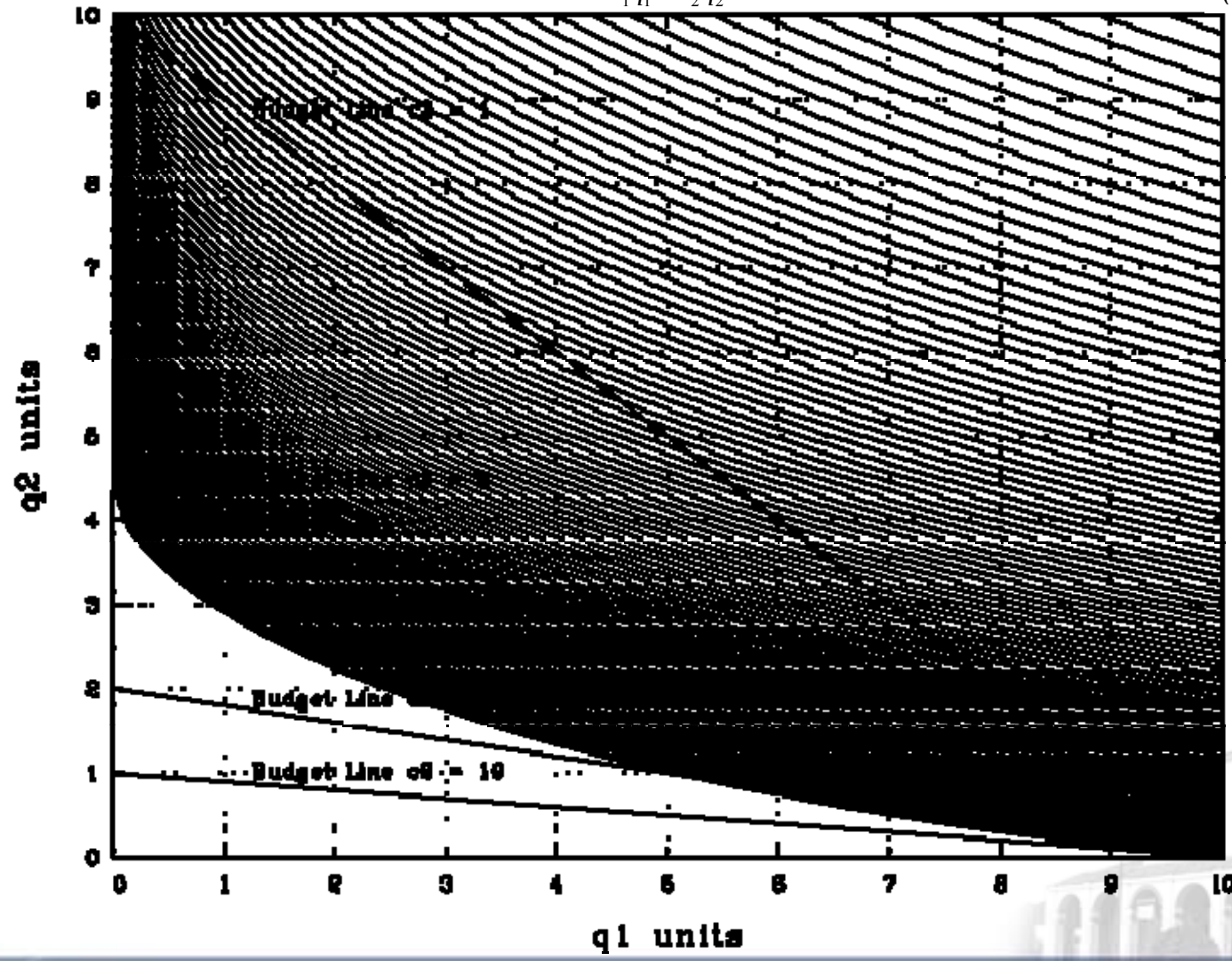


The new system is affordable if there are (q_1, q_2) combinations for which

$$v(q_1, q_2) \geq v(10,0) = v_1(10) = 0.492 \tag{3}$$

and

$$c_1 q_1 + c_2 q_2 \leq 10. \tag{4}$$



$N =$ the number of grid points satisfying eq.(3)

$M =$ the number of grid points that lay on or below the respective budget line

$A = N / M$ is an estimate of the measure of affordability

TABLE 1: AFFORDABILITY MEASURE

c_2	q_2	$A^{(1)}$	$A^{(0.1)}$	$A^{(0.01)}$
1.00	10	0.733	0.736	0.753
2.00	5	0.400	0.474	0.507
2.50	4	0.259	0.348	0.384
3.33	3	0.100	0.242	0.215
5.00	2	0.063	0.047	0.054
10.00	1	0.000	0.002	0.001

$A^{(x)}$ denotes the value of A obtained using a grid of width x.

Investigating the effects of varying system parameters besides the new system cost.

TABLE 2: AFFORDABILITY MEASURE $(a_2 = 0.31)$

c_2	q_2	$A^{0.01}$
1.00	10	0.848
2.00	5	0.696
2.50	4	0.620
3.33	3	0.494
5.00	2	0.260
10.00	1	0.020

The new system now possesses a modest affordability for $c_2 = 5.0$

Affordability-Effectiveness Example

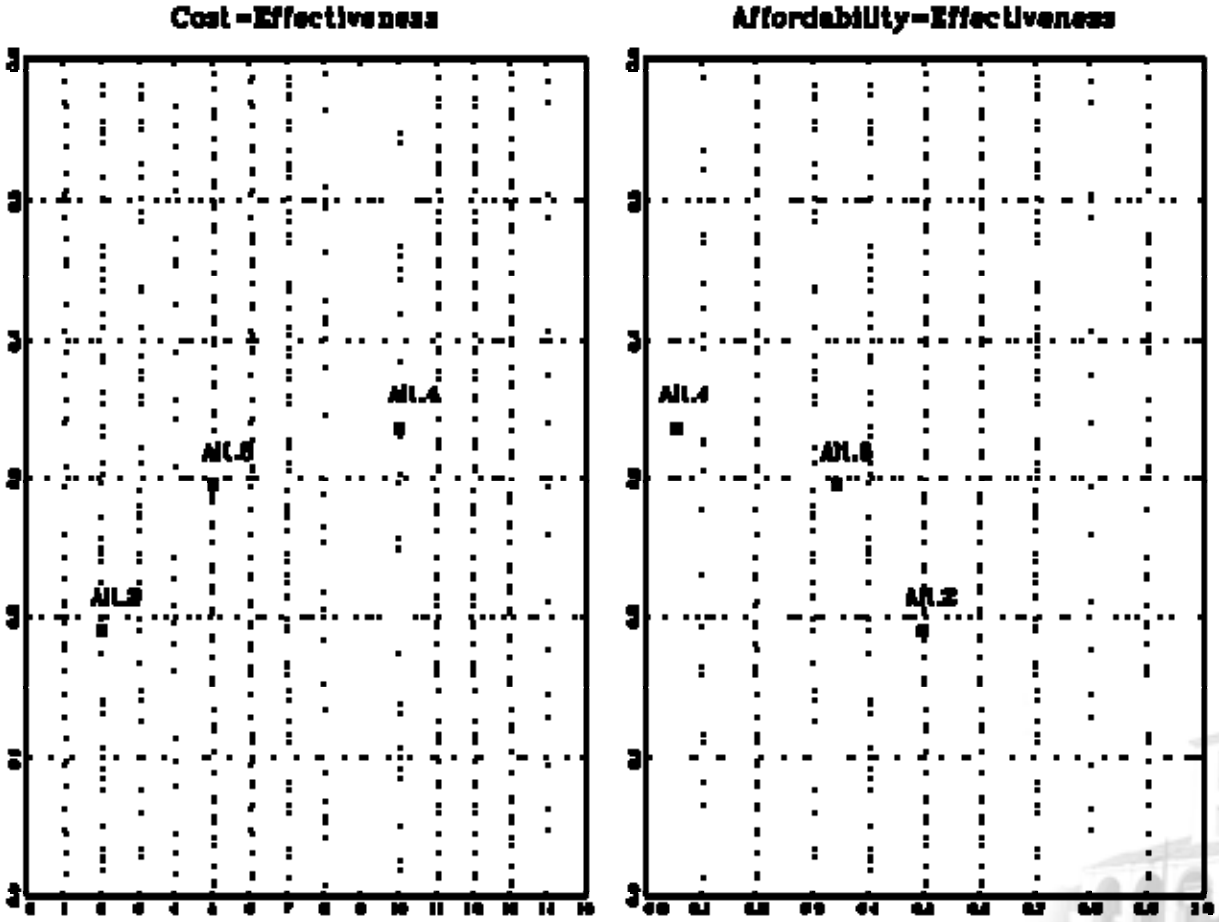
TABLE 3: Multiple Candidate System Example

k	a_k	b_k	c_k
1	0.17	0.6	1.0
2	0.21	0.8	2.0
3	0.35	0.7	5.0
4	0.41	0.9	10.0

$c_2 = 2.0$ $A_2 = 0.495$

$c_3 = 5.0$ $A_3 = 0.343$

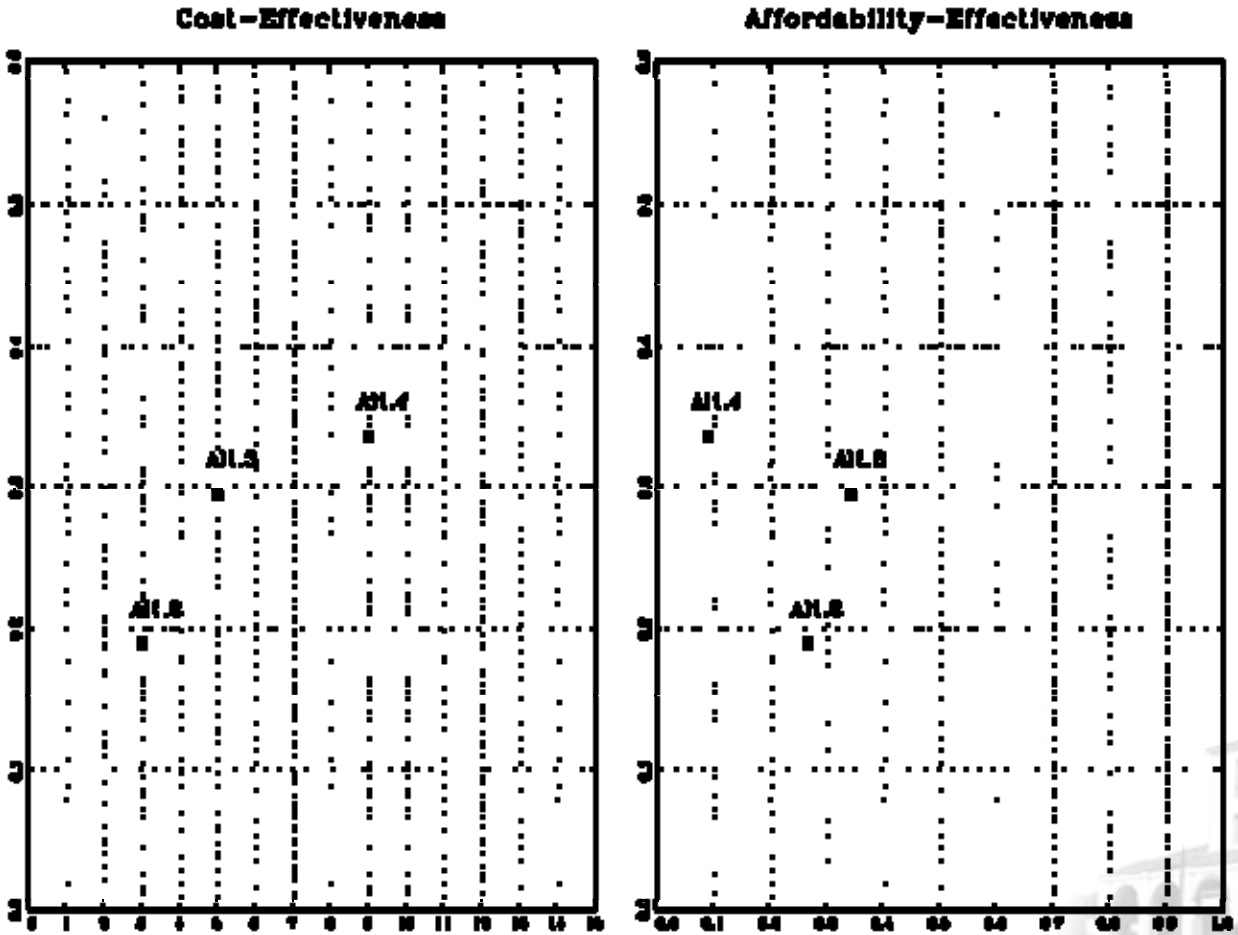
$c_4 = 9.0$ $A_4 = 0.054$



$c_2 = 3.0$ $A_2 = 0.26$

$c_3 = 5.0$ $A_3 = 0.33$

$c_4 = 9.0$ $A_4 = 0.09$



AFFORDABILITY RISK

Interpret (q1, q2) ϵ $\mathcal{N}$ as an *event*.

$P\{\mathcal{N} \neq 0\} = 1 - P\{\mathcal{N} = 0\}$

Interpret $A = N/M$ as a *random variable*.

P 0 1 P 0

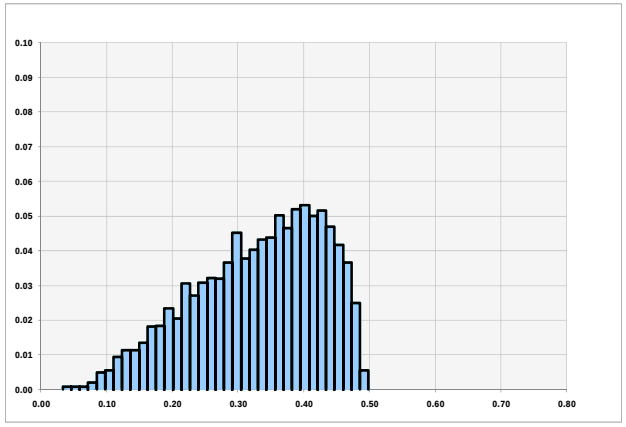
- AFFORDABILITY RISK (Type 1): *The likelihood that an alternative is unaffordable*:
$$P(A = 0).$$
- AFFORDABILITY RISK (Type 2): *The probability that the measure of affordability is less than some minimally acceptable level*:
$$P(A < \alpha_{\min}).$$

Affordability Risk Example

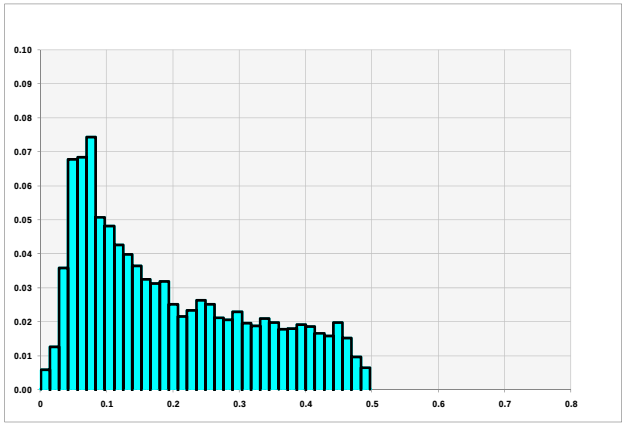
TABLE 4: SIMULATION SCENARIOS

RUN	a_2	b_2	c_2	d	B
1	0.21	0.80	PERT (2.0,2.2,5)	.3	PERT (9,10,10.05)
2	0.21	0.80	Uniform (2,5)	.3	PERT (9,10,10.05)
3	0.21	0.80	Uniform (2,5)	.3	Uniform (9,10.05)
4	PERT (.17,.21,.22)	PERT (.65,.80,.85)	Uniform (2,5)	.3	Uniform (9,10.05)
5	PERT (.17,.21,.22)	PERT (.65,.80,.85)	Uniform (2,5)	.3	Uniform (8,10.05)
6	PERT (.20,.35,.40)	PERT (.65,.80,.85)	Uniform (2,5)	.3	Uniform (8,10.05)

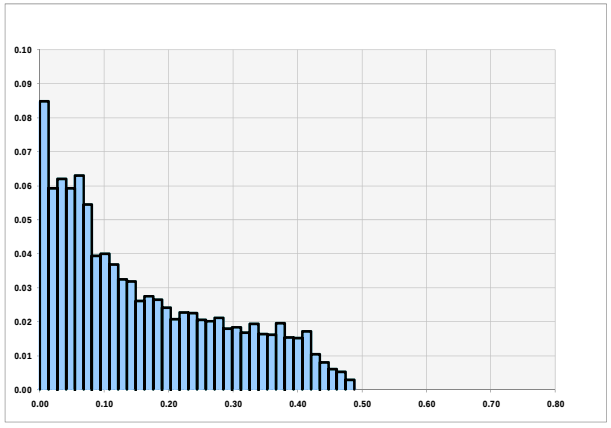
5000 trial using Roman hypercube sampling



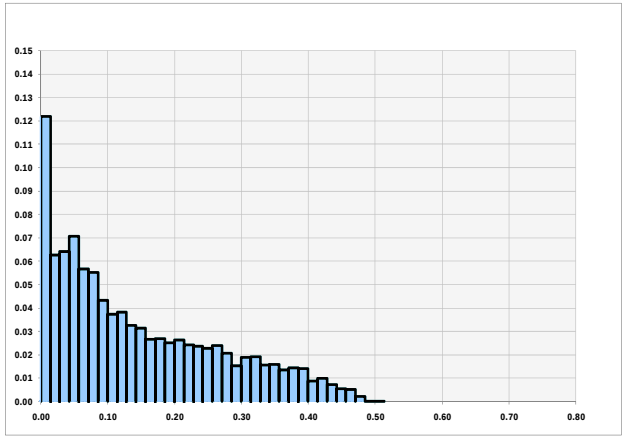
Run 1: Cost and Budget Uncertainty



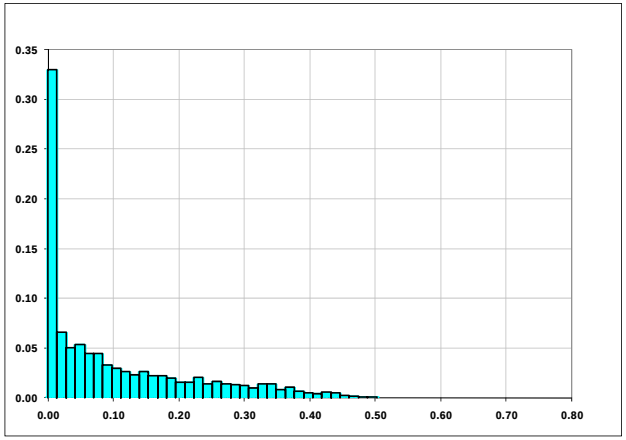
Run 2: Cost and Budget Uncertainty



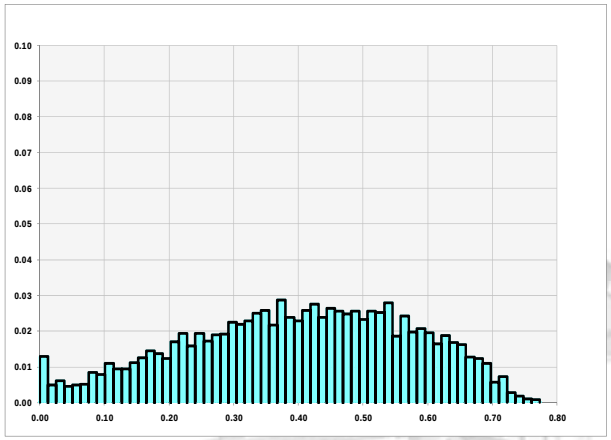
Run 3: Cost and Budget Uncertainty



Run 4: Effectiveness, Cost and Budget



Run 5 : Effectiveness, Cost and Budget



Run 6 : Effectiveness, Cost and Budget

Affordability Risk Example

5000 trial using Roman hypercube sampling

TABLE 5: AFFORDABILITY STATISTICS

RUN	$\alpha_{(.025)}$	$\alpha_{(.975)}$	$P(A=0)$	$P(A \le 0.1)$
1	0.125	0.475	0.000	0.010
2	0.026	0.475	0.000	0.298
3	0.000	0.467	0.023	0.343
4	0.000	0.452	0.030	0.355
5	0.000	0.425	0.211	0.502
6	0.061	0.693	0.003	0.041