An Approximate Dynamic Programming (ADP) Approach for Weapon System Financial Execution Management

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Monthly Financial Commitment Questions

Sequential Decision Making Problem

Oct	Nov	Dec	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep
x_1	x_2	…	…	…	x_{t-1}	x_t	x_{t+1}	…	…	…	x_n

Decision Makers:

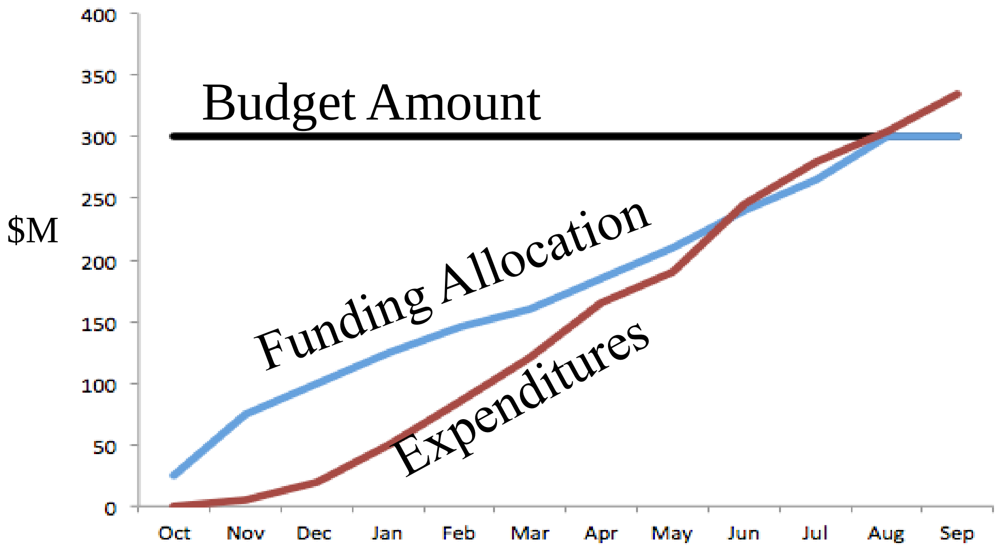
? ? $ $? $

DXC.technology
LOCKHEED MARTIN
BOEING
IDA
NORTHROP GRUMMAN
ALATEC Inc
DISA
DEFENSE INFORMATION SYSTEMS AGENCY
DEPARTMENT OF DEFENSE
Raytheon
Booz | Allen | Hamilton
strategy and technology consultants
CACI
EVER VIGILANT
MITRE
IBM
SPAWAR
APL
The Johns Hopkins University
APPLIED PHYSICS LABORATORY
U.S. ARMY CECOM
One Vision, One Mission-The Warfighter.

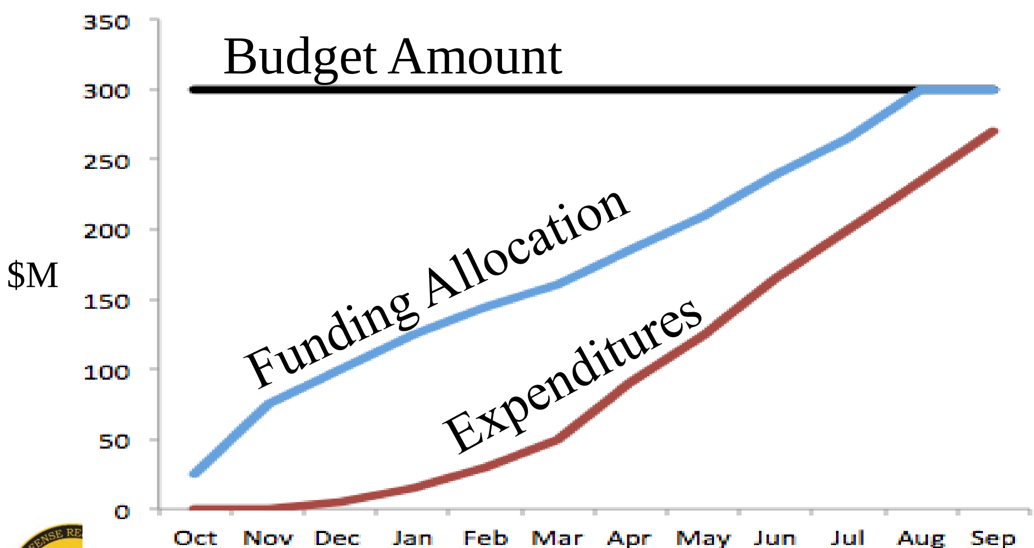
Which Contractors?

How Much?

Two Traditional Problems



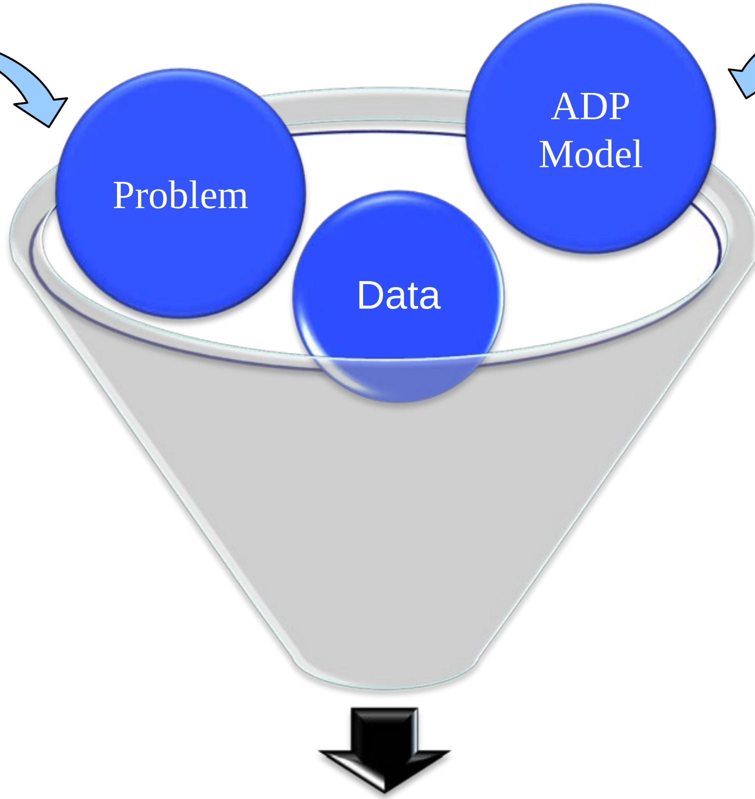
1) Overrun



2) Under Utilized Budget

Methodology

Make the Best Cash Allocation Decisions to Maximize the Utilization of My Budget Without Causing an Overrun



Stages
States
Actions / Decision
Pre-Decision State
Post-Decision State
Uncertainty / Exogenous Data
Learning Factor
Exploration / Exploitation

Reinforcement Learning
- Q Learning
- Value Function Learning

Dynamic Model Responsive to Changes in Expenditure Environment

ADP Cash Allocations are Conservative, Holds Back 2% to 7%
- Identifies Unutilized Funds Earlier in FY

Stages of a Transaction

Budget /
Programmed Funds (P)

COMMITMENT (C)
Administrative
Reservation of Funds

Authorization Forms
(FAD, MIPR, RDD)
Distribution of Funds

OBLIGATION (O)
Legal Reservation of
Funds

ACCRUALS (A)
Evidence of Goods /
Services Provided
(Receiving Report)

EXPENDITURE (E)
Disbursement
Payment for Goods or
Services

FAD – Fund Authorization Document
MIPR – Military Interdepartmental Purchase Request
RDD – Resource Distribution Document

State-Space Vector

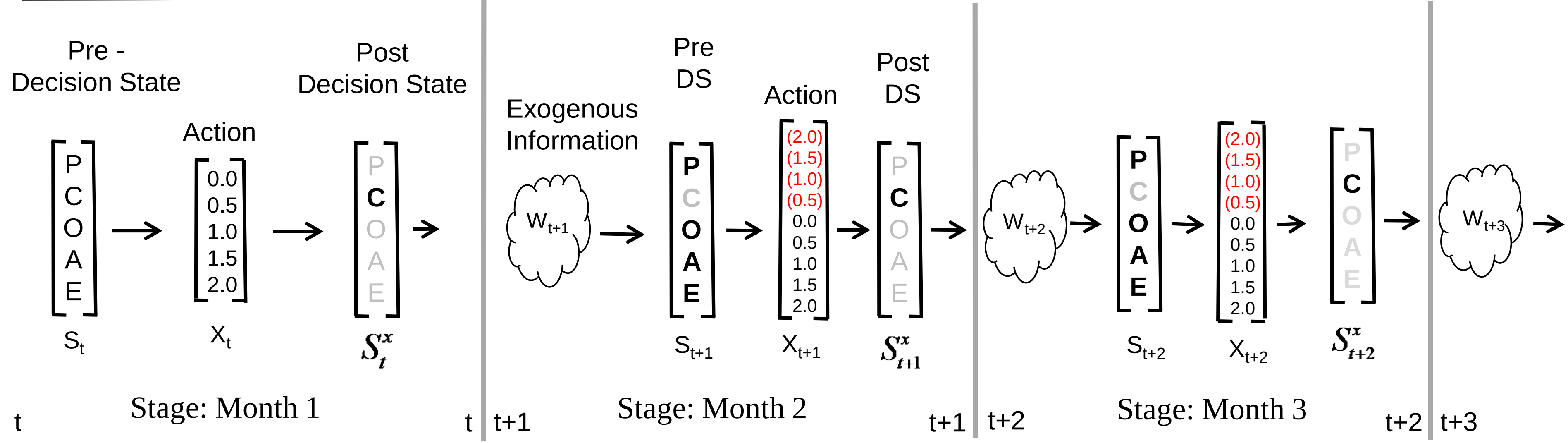
$$S_t = \begin{array}{l} \text{Programmed} \\ \text{Commitments} \\ \text{Obligations} \\ \text{Accruals} \\ \text{Expenditures} \end{array} \begin{bmatrix} P \\ C \\ O \\ A \\ E \end{bmatrix}$$

Defines the Monthly State-Space of My Budget

$\begin{bmatrix} P \\ C \\ O \\ A \\ E \end{bmatrix}_1$	$\begin{bmatrix} P \\ C \\ O \\ A \\ E \end{bmatrix}_2$	$\longrightarrow$			$\begin{bmatrix} P \\ C \\ O \\ A \\ E \end{bmatrix}_{t-1}$	$\begin{bmatrix} P \\ C \\ O \\ A \\ E \end{bmatrix}_t$	$\begin{bmatrix} P \\ C \\ O \\ A \\ E \end{bmatrix}_{t+1}$	$\longrightarrow$			$\begin{bmatrix} P \\ C \\ O \\ A \\ E \end{bmatrix}_T$
1	2	…	…	…	t-1	t	t+1	…	…	…	T
Oct	Nov	Dec	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep

Stages

Simulation Process (Value Function Learning)



Initial

Post Decision State	$\overline{V}_t(S_t^x)$
[P,C,O,A,E]	0.0
[P,C,O,A,E]	0.0
[P,C,O,A,E]	0.0
[P,C,O,A,E]	0.0
[P,C,O,A,E]	0.0
[P,C,O,A,E]	0.0
[P,C,O,A,E]	0.0
[P,C,O,A,E]	0.0
[P,C,O,A,E]	0.0
[P,C,O,A,E]	0.0
[P,C,O,A,E]	0.0
[P,C,O,A,E]	0.0
[P,C,O,A,E]	0.0
[P,C,O,A,E]	0.0
[P,C,O,A,E]	0.0

Updated

Post Decision State	$\overline{V}_t(S_t^x)$
[P,C,O,A,E]	0.66
[P,C,O,A,E]	0.36
[P,C,O,A,E]	0.97
[P,C,O,A,E]	0.47
[P,C,O,A,E]	0.32
[P,C,O,A,E]	0.03
[P,C,O,A,E]	0.32
[P,C,O,A,E]	0.55
[P,C,O,A,E]	0.54
[P,C,O,A,E]	1.80
[P,C,O,A,E]	0.02
[P,C,O,A,E]	0.03
[P,C,O,A,E]	0.78
[P,C,O,A,E]	0.45
[P,C,O,A,E]	0.37

Learning Factor Calculation $\overline{V}(S^x)$

Start by initializing all $\overline{V}(S^x)$

'Learning' Step 1:

Use Bellman equation to observe and collect state-space values

$$\hat{v}_t^n(S_t^n) = Min_{x_t}[(C(S_t^n, x_t) + \gamma \overline{V}_t^{n-1}(S_t^{n,x})]$$

Sample Value = $\hat{v}_t^n(S_t^n)$

'Learning' Step 2:

Use stochastic approximation to smooth or learn state values

$$\underbrace{\overline{V}_{t-1}^{n}(S_{t-1}^{x,n})}_{\text{update the value of the previous state}} = (1-\alpha_{n-1})\underbrace{\overline{V}_{t-1}^{n-1}(S_{t-1}^{x,n})}_{\text{current value of the previous state}} + \alpha_{n-1}\underbrace{\hat{v}_t^n(S_t^n)}_{\text{sample value}}$$

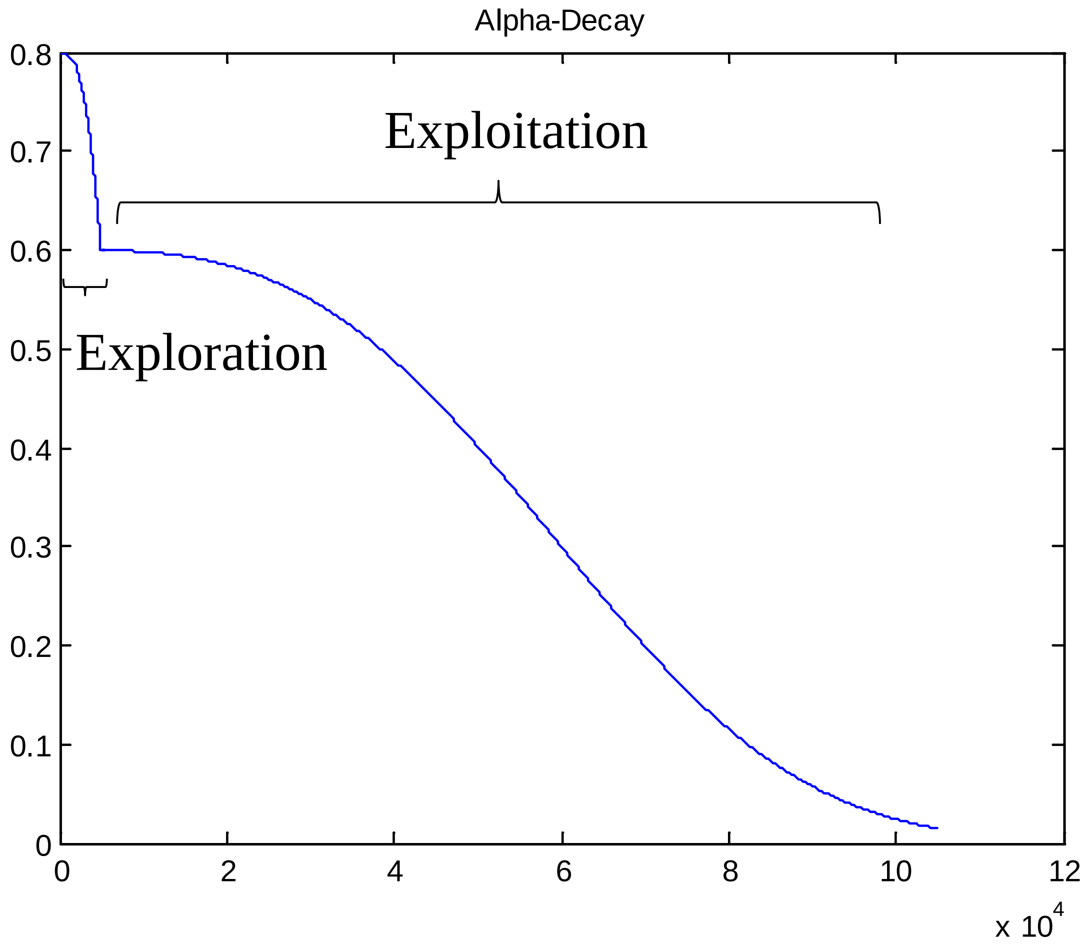
Learning Factor = $\overline{V}_{t-1}^{n}(S_{t-1}^{x,n})$

Objective Function: Learnt Model

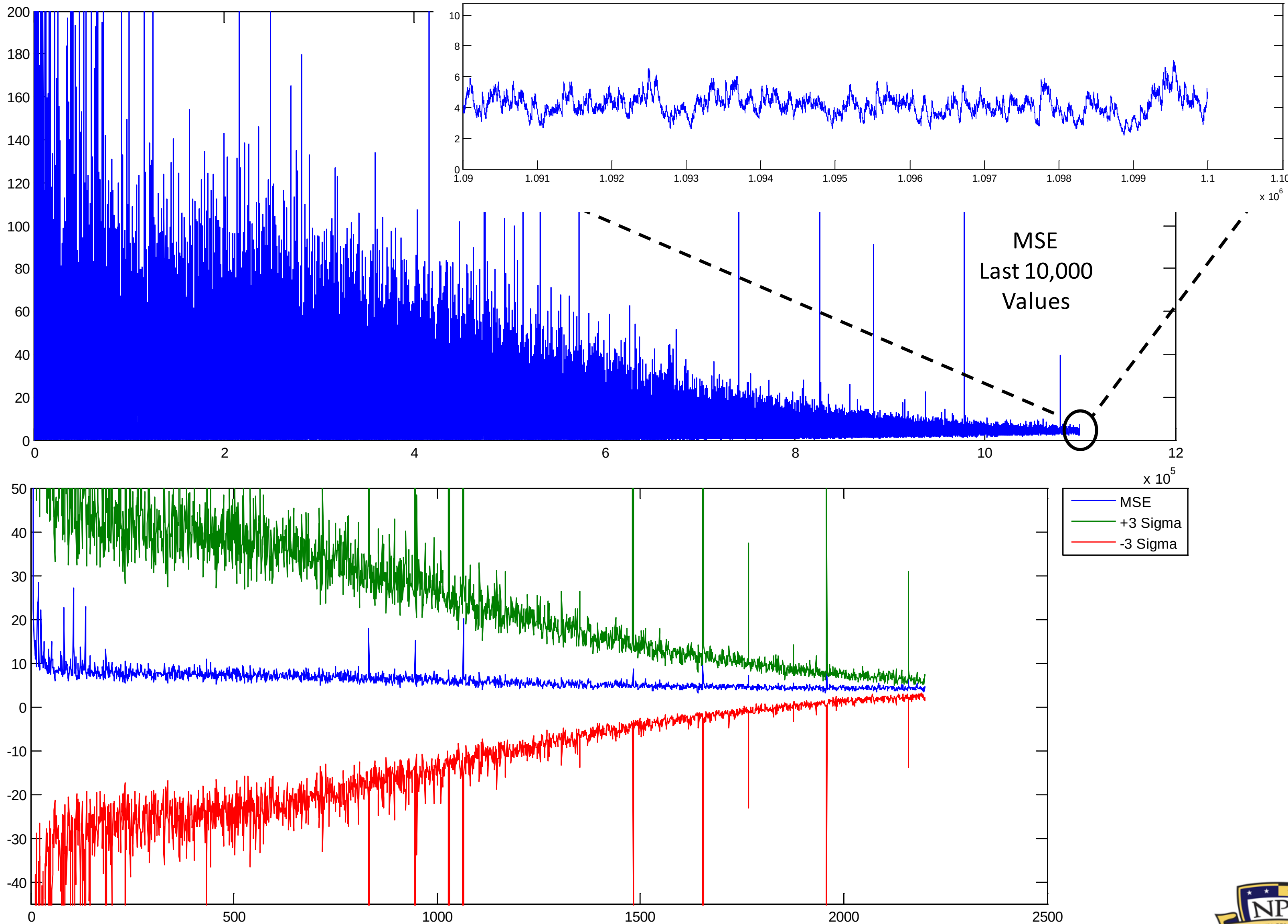
Use Bellman equation to make a decision

$$x_t^*(S_t) = \arg\min_{x_t \in X_t}(C_t(S_{t,}x_t) + \underbrace{\gamma \overline{V}_t(S_t^x)}_{\text{learning factor}})$$

Step-Size Properties



Examples of MSE Convergence



Reducing Dimensionality

- Exorbitant run-times led to reducing the state-space dimensionality

1) $$\begin{bmatrix}P\\C\\O\\A\\E\end{bmatrix} \rightarrow \begin{bmatrix}C\\E\end{bmatrix}$$

Action Parameter (C)

Predictive Parameter (E)

2) Added Conditional $C \geq E$

Before

$\begin{bmatrix}P\\C\\O\\A\\E\end{bmatrix}$

$5.0M project
$0.250M increments
$21^5 \approx$ 4 million state-spaces

After

$\begin{bmatrix}C\\E\end{bmatrix}$

$5.0M project
$0.250M increments
$21^2 = 441$ state-spaces
with conditional → 231 state-spaces

Two Policy Approaches

What Amount of $$$ Do We Give Our Contractor This Month?

Estimated Requirement \$15M
<u>Already Committed \$10M</u>
Fund \$5M

Stubby Pencil (myopic) Approach

Decision Policy

$$x_t^*(S_t) = \arg\min_{x_t \in X_t}(C_t(S_t, x_t))$$

Estimated Requirement \$15M
<u>Already Committed \$10M</u>
Best {Commitment Action + PDS Learning Factor}

ADP Algorithm Approach

Decision Policy

$$x_t^*(S_t) = \arg\min_{x_t \in X_t}(C_t(S_t, x_t) + \gamma \overline{V}_t(S_t^x))$$

Test Cases

Test Cases

<u>Q - Learning</u>

Test Case #1 Trial #1, Trail #2, Trail #3	**2 Projects → $7M** Tested: different iteration counts / alpha-decay start & stop points
Test Case #2 Trial #1	**3 Projects → $22M** Tested: added projects / budget
Test Case #3 Trial #1	**4 Projects → $68M** Tested: added projects / budget / months (24)

<u>Value Function Learning</u>

Test Case #1 Trial #1, Trail #2	**2 Projects → $7M - $9.5M** Tested: budget size / uncertainty / months (16)
Test Case #2 Trial #1, Trial #2, Trial #3	**3 Projects → $22M** Tested: different iteration counts / alpha-decay start & stop points
Test Case #3 Trial #1, Trial #2, Trial #3, Trial #4. Trial #5, Trial #6	**3 Projects → $22M** Tested: different iteration counts / alter initial funding plans alpha-decay start & stop points
Test Case #4 Trial #1	**4 Projects → $44M** Tested: added projects / budget / uncertainty

Test Case #2 – Trial #1

Number of Projects: 3
Number of Months: 12
55,000 Simulation iterations
State-space size = 4,005

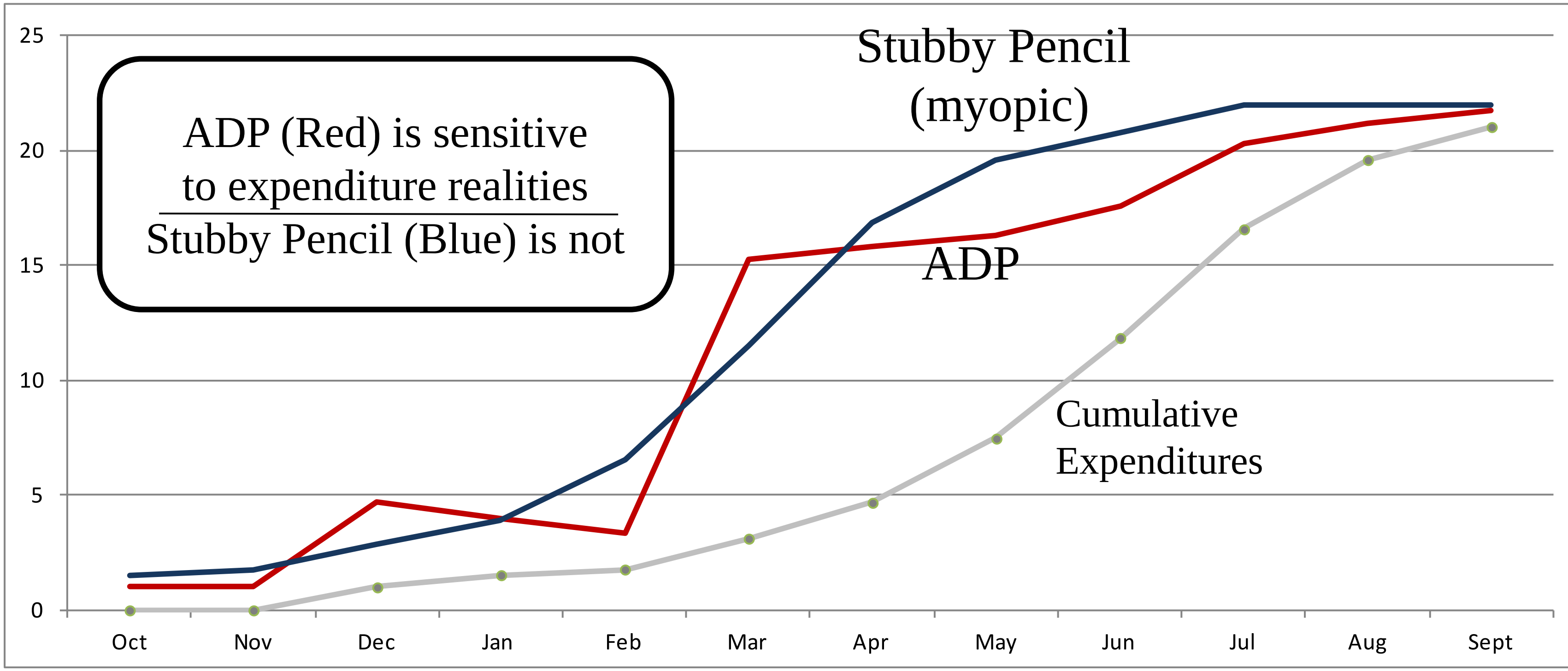
RunTime: 11hrs 18Minutes
2.33 GHz Intel® Xeon
3.00 GB RAM

Fluctuating Expenditure Planning Figures

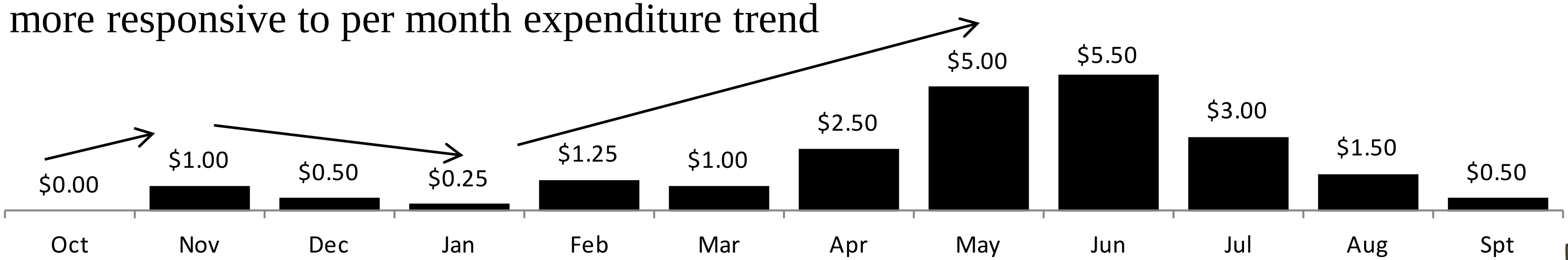
$M

		Project #1		Project #2		Project #3		Total	
t	month	Comm	Exp	Comm	Exp	Comm	Exp	Comm	Exp
1	Oct			1.500				1.500	
2	Nov			0.250	1.000			0.250	1.000
3	Dec			0.250	0.500	1.000		1.250	0.500
4	Jan				0.250	1.000		1.000	0.250
5	Feb	0.500			0.250	2.000	1.000	2.500	1.250
6	Mar	1.000				4.000	1.000	5.000	1.000
7	Apr	1.500	0.500			4.000	2.000	5.500	2.500
8	May	1.000	1.000			2.000	4.000	3.000	5.000
9	Jun	0.500	1.500			1.000	4.000	1.500	5.500
10	July	0.500	1.000				2.000	0.500	3.000
11	Aug		0.500				1.000		1.500
12	Sept		0.500						0.500
		5.000	**5.000**	**2.000**	**2.000**	**15.000**	**15.000**	**22.000**	**22.000**

Test Case #2 – Trial #1 (Commitment Policies)



ADP more responsive to per month expenditure trend



Test Case #3 – Trial #2

Number of Projects: 3
Number of Months: 12
80,000 Simulation iterations
State-space size = 4,005

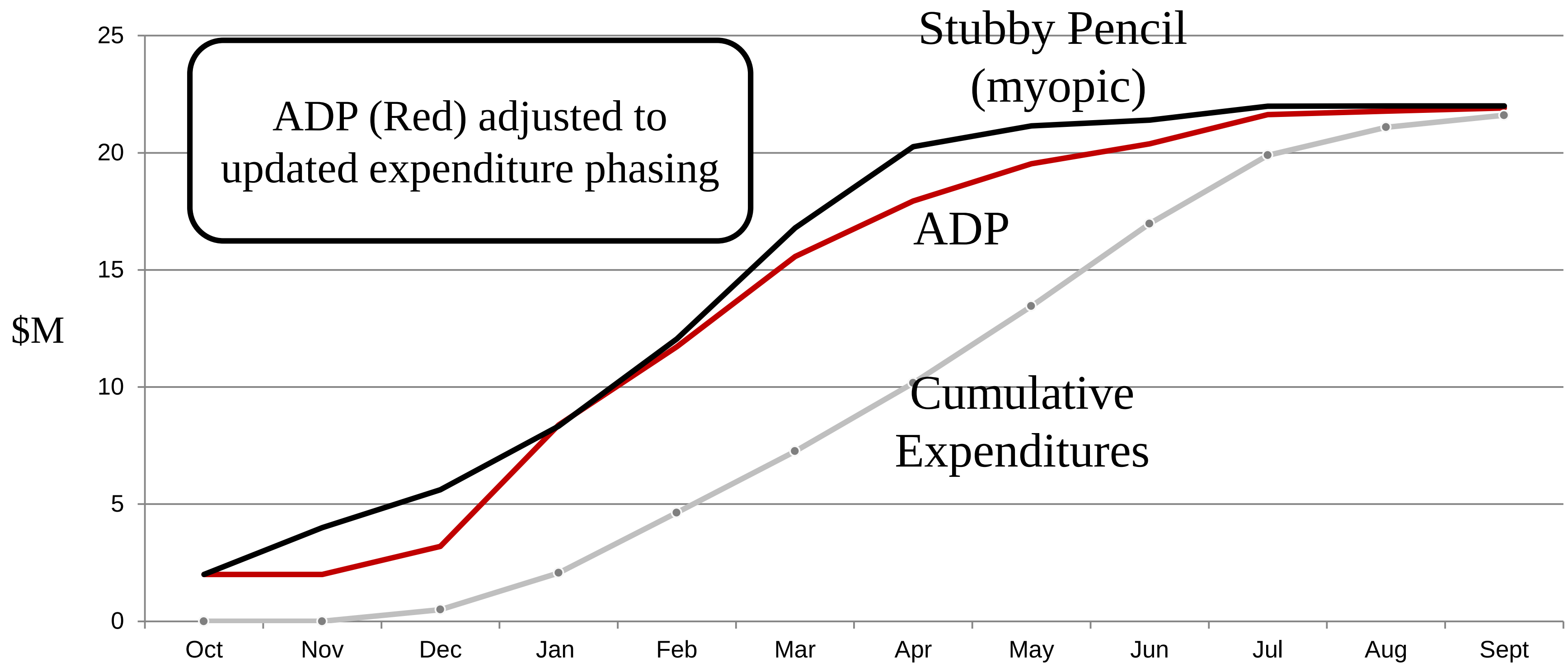
RunTime: 17hrs 31Minutes
2.33 GHz Intel® Xeon
3.00 GB RAM

Smoothed Expenditure Planning Figures

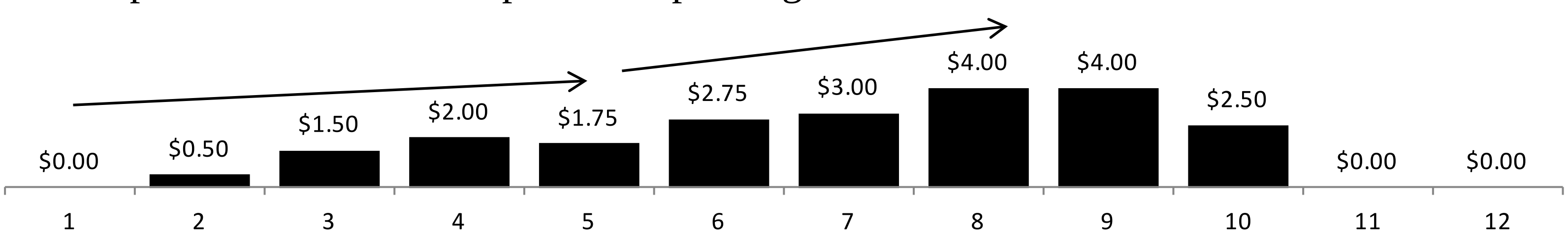
$M

		Project #1		Project #2		Project #3		Total	
t	month	Comm	Exp	Comm	Exp	Comm	Exp	Comm	Exp
1	Oct			1.000		1.000		2.000	
2	Nov	0.500		0.500	0.500	1.000		2.000	0.500
3	Dec	0.500		0.250	0.500	1.000	1.000	1.750	1.500
4	Jan	0.500	0.500	0.250	0.500	2.000	1.000	2.750	2.000
5	Feb	1.000	0.500		0.250	2.000	1.000	3.000	1.750
6	Mar	1.000	0.500		0.250	3.000	2.000	4.000	2.750
7	Apr	1.000	1.000			3.000	2.000	4.000	3.000
8	May	0.500	1.000			2.000	3.000	2.500	4.000
9	Jun		1.000				3.000		4.000
10	July		0.500				2.000		2.500
11	Aug								
12	Sept								
		5.000	**5.000**	**2.000**	**2.000**	**15.000**	**15.000**	**22.000**	**22.000**

Test Case #3 – Trial #2 (Commitment Policies)

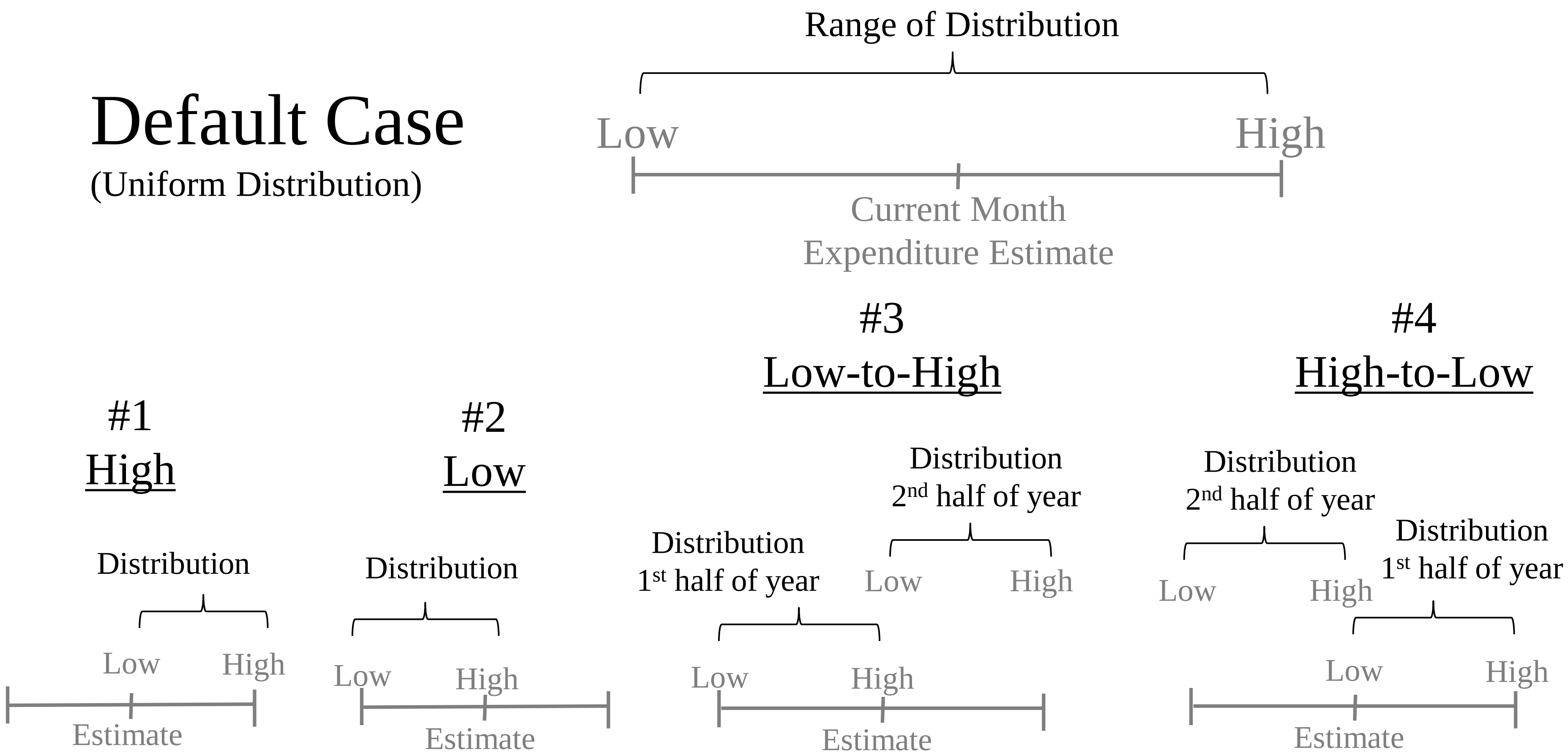


Updated incremental expenditure phasing

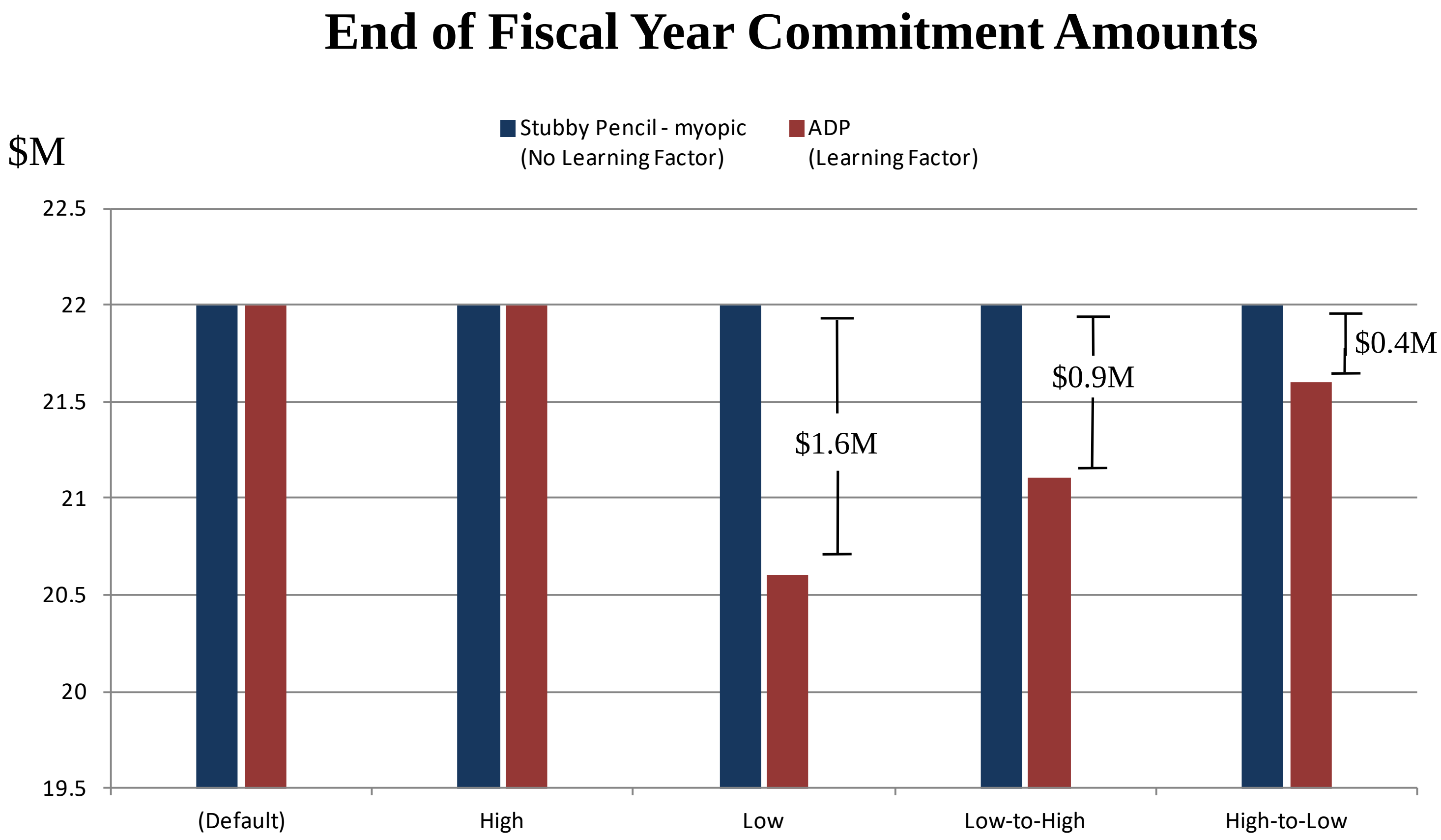


Sensitivity Analysis

Examines policy responses given four types of uncertainty expenditure scenarios



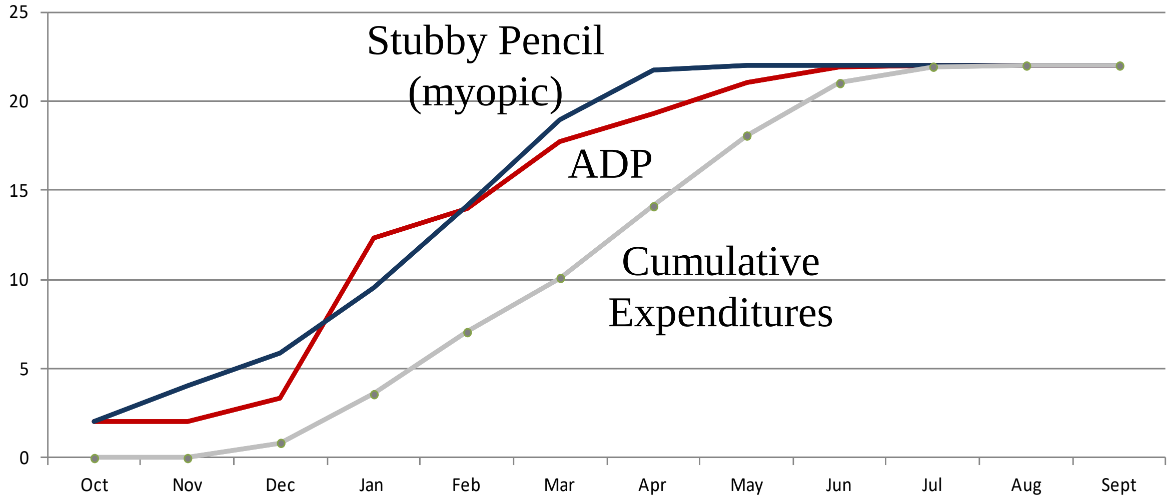
Observations



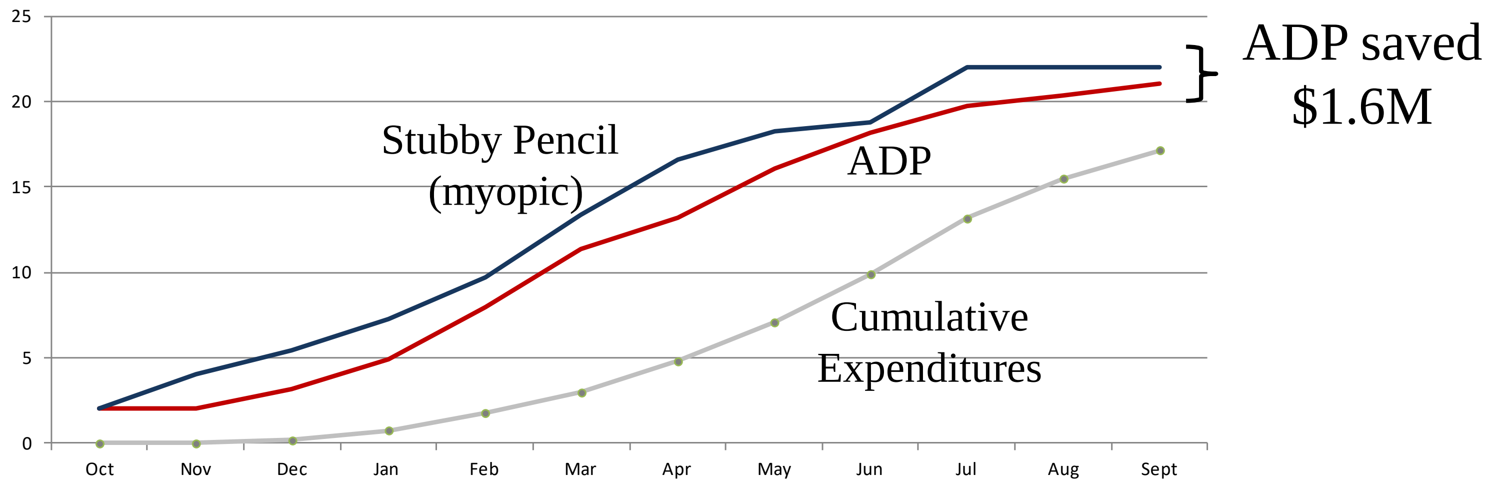
- In 3 of 5 cases ADP did not commit full $22M budget
  - Represent additional work opportunities

Sensitivity Analysis

High Expenditures



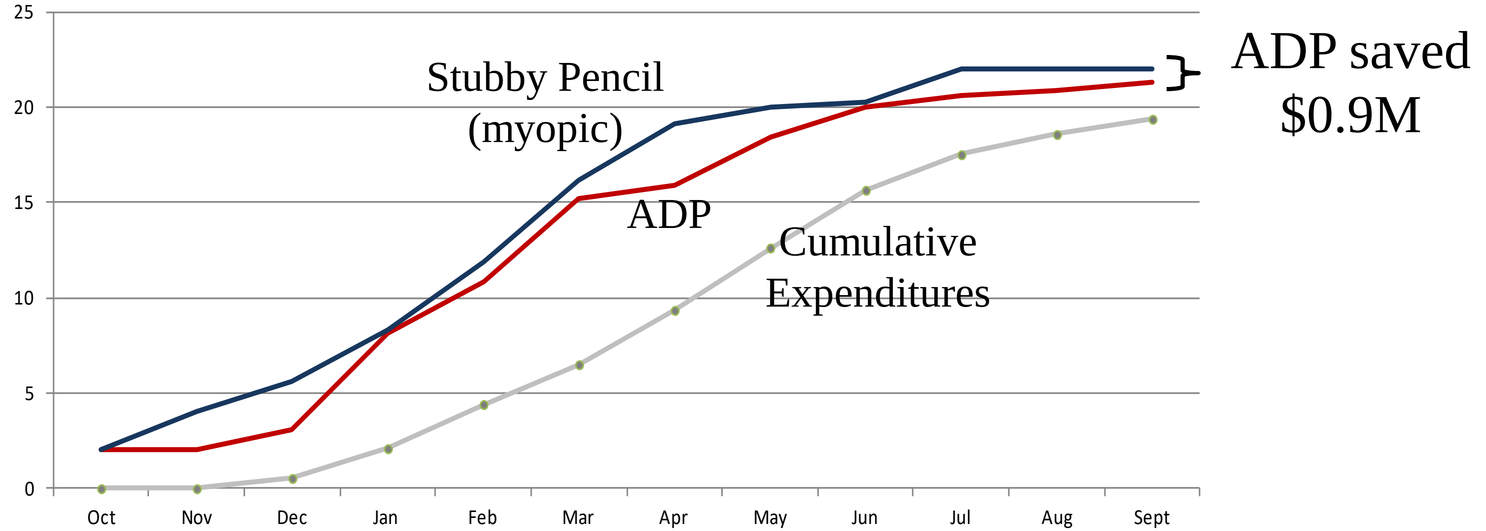
Low Expenditures



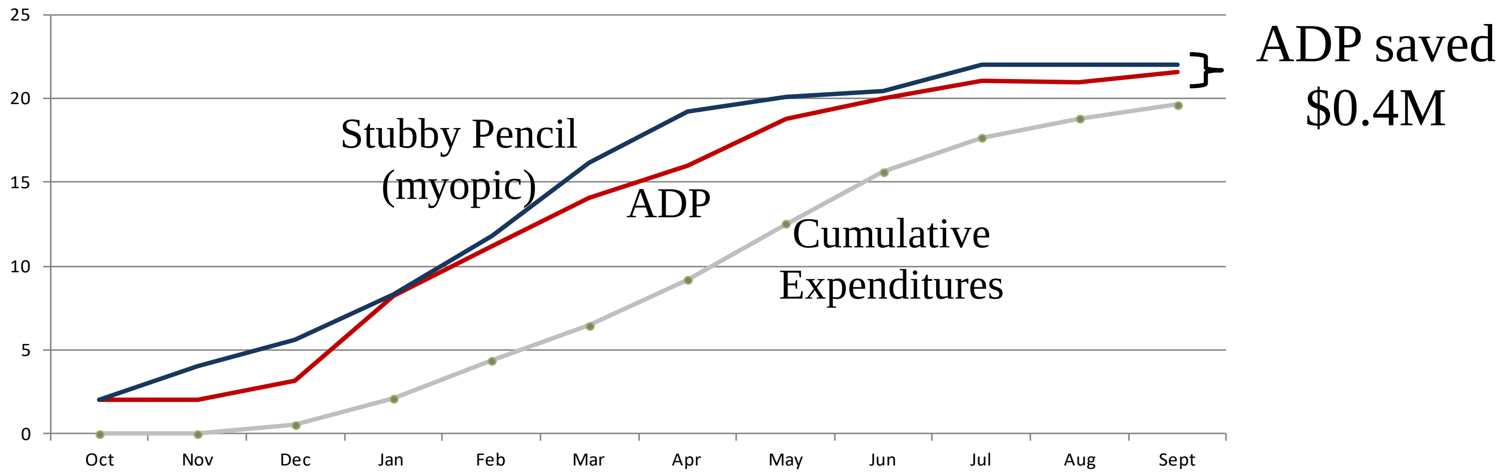
ADP produces more conservative policies throughout FY

Sensitivity Analysis

Low-to-High Expenditures

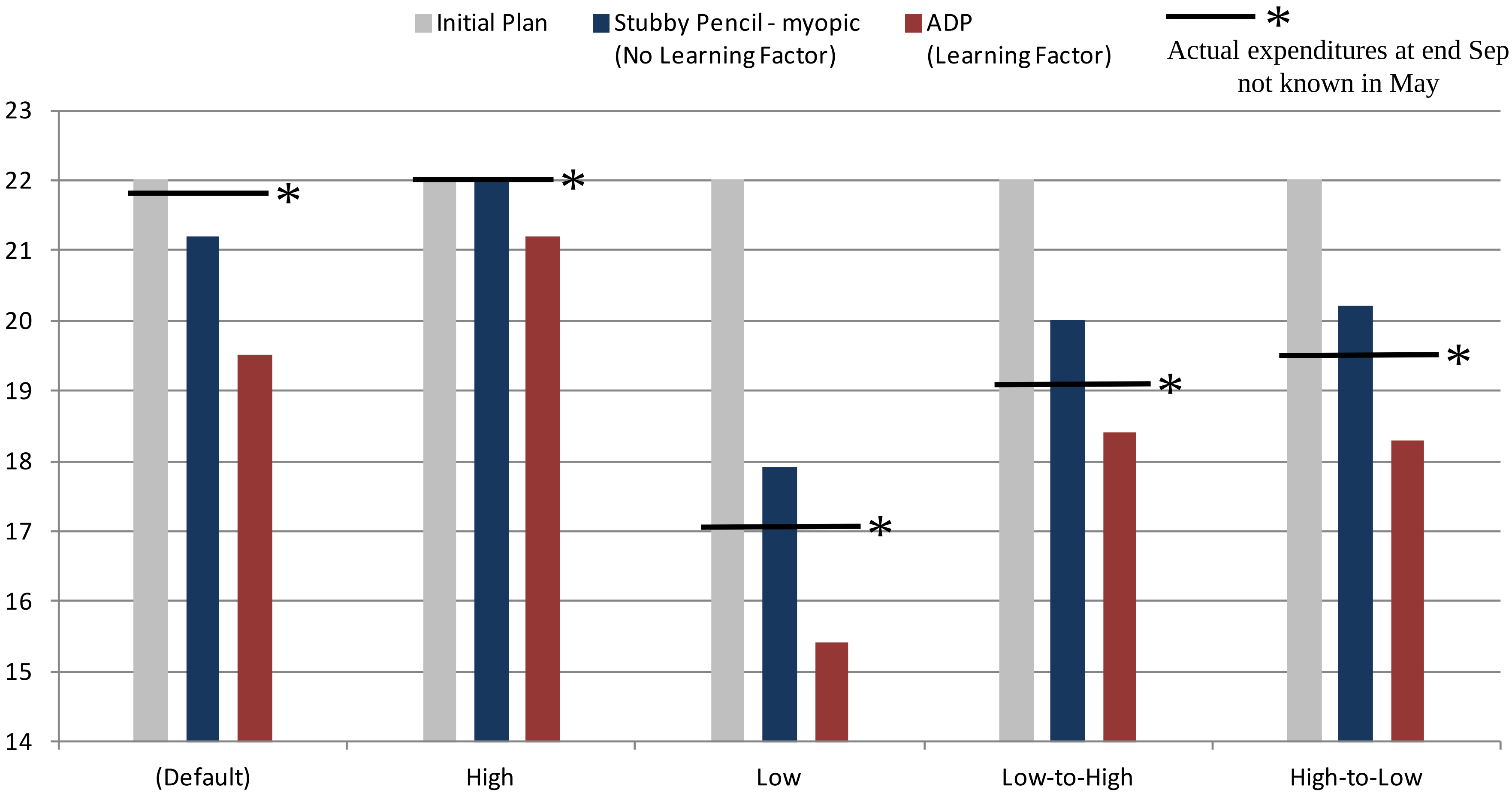


High-to-Low Expenditures



ADP produces more conservative policies throughout FY

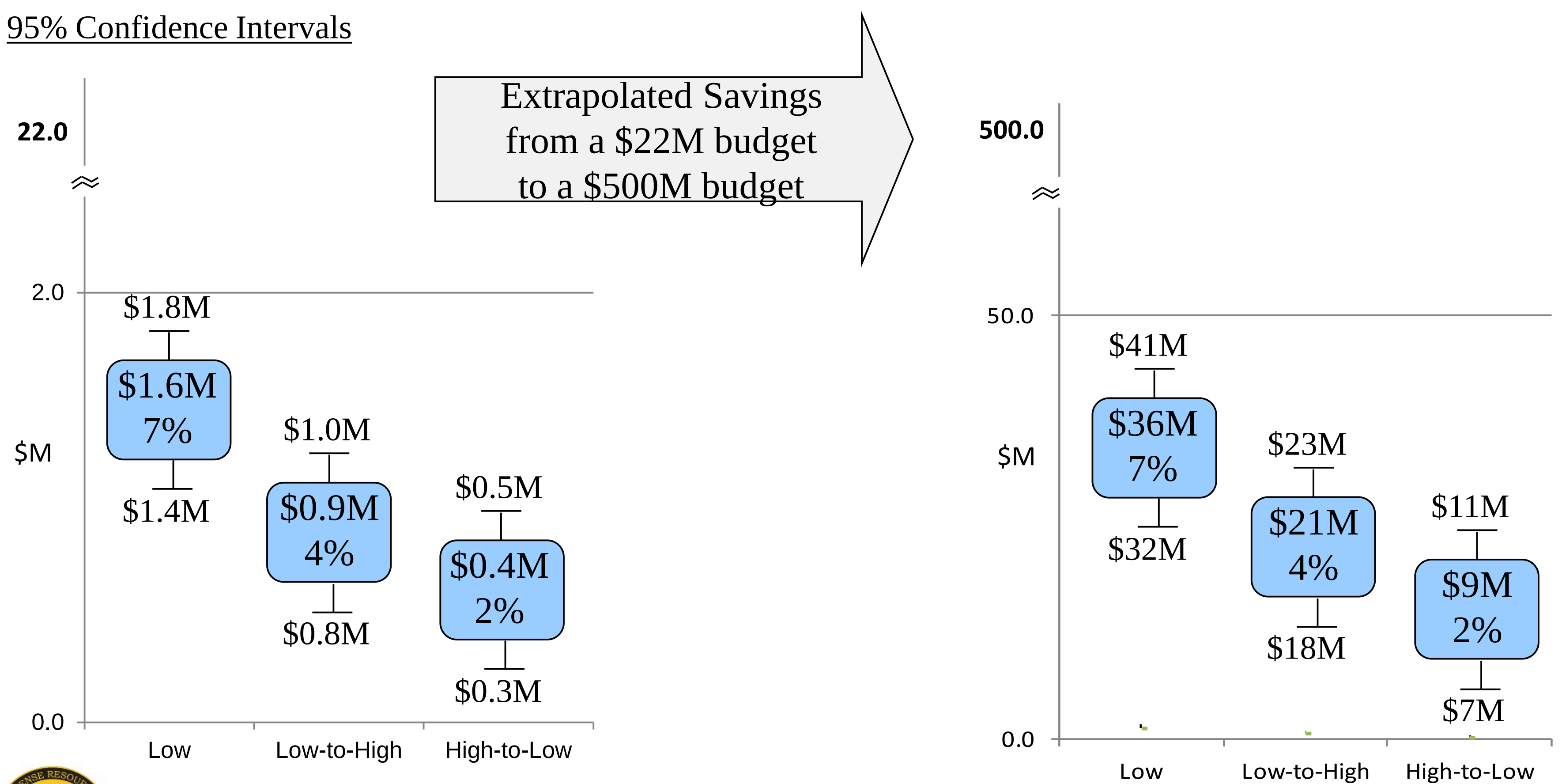
End of Month May Commitment Levels



- Over-commitments represent lost work opportunities

Implications and Inferences

Scalability: Bigger Budgets are Likely to Reveal Significant Savings



Conclusion

Financial Execution using Approximate Dynamic Programming (ADP)

- Simulation Driven Reinforcement Learning Algorithm
- Designed for Stochastic Sequential Decision Making Problems
- Recommends More Conservative Cash Allocations
- If Scalable, Potential to Identify Significant Savings

Areas for Continued Research

- Scalability
- Exogenous Information
- Lag-Time on Reporting
- Continuing Resolution Authority
- Color-of-Money

QUESTIONS

ADP Concepts

State-Space
- Discretized state-space for each project
- Aggregate Vs. Incremental Data

Exploration Vs. Exploitation – 'Learnt' Phase
- Difference Between the Two
- How Many Simulation Iterations Do You Dedicate to One or the Other

Exogenous Data
- Currently as a Uniform Distribution

Step-Size Choice
- Adapted Deterministic Harmonic alpha-decay

Structure of Objective Function (Bellman's Equation)

Stopping Criteria
- MSE

Curse of Dimensionality
Curse of Modeling

Pre-Decision Vs. Post-Decision State Learning

Formulation of Bellman's Equation Around the Post-Decision State Variable

Look-Up Tables
- Q-Learning
- Value Learning
- Q-Learning around Post-Decision State

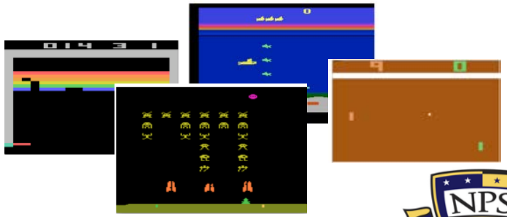
Where is ADP Being Used Today

Aircraft Inventory Levels Simao and Powell
Taxi-out Time of a Flight Ganesan, Poornima, Sherry

Norfolk Southern Locomotive Planning Powell, Bouzaiene-Ayari, et al.
Schneider's 5,000 Fleet Management Simao, Day, et al.

Optimal Blood Inventory Management Cant
Kidney Exchange w/Transplant Chains Dickerson, Procaccia, Sandholn

Playing Atari w/Deep Reinforcement Learning DeepMind Technologies



Vernacular Used by Different Communities

Simulation based optimization method used to solve sequential decision making problems in environments of uncertainty

Operations Research – Approximate/Adaptive Dynamic Programming

Control Theory – Neuro-Dynamic Programming

Artificial Intelligence – Reinforcement Learning

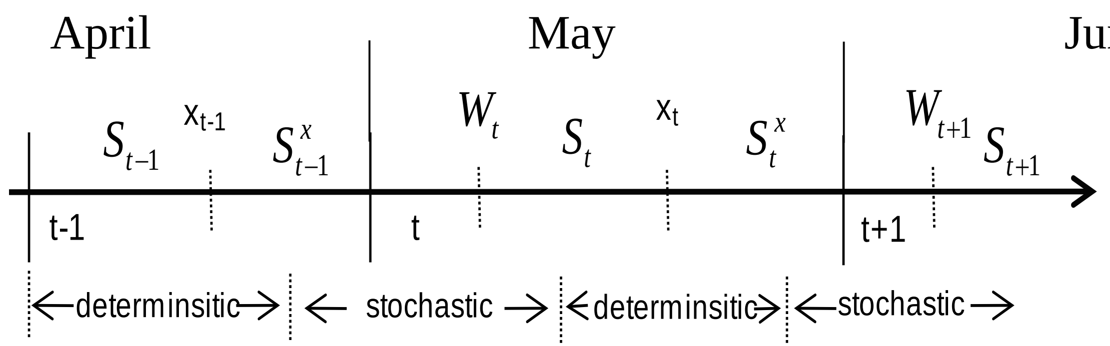
Objective Function

Expectation Form of Bellman's Equation

$$V_t(S_t) = min_{x_t \in X_t}(\underbrace{C_t(S_t, x_t)}_{\text{abs |delta 3 months|}} + \gamma \underbrace{E\{V_{t+1}(S_{t+1})|S_t\}}_{\text{Use ADP to 'learn' values}})$$

$$C_t(S_t^n, x_t) = \left\{ \text{ABS} \left| \begin{array}{c}\text{Cumulative}\\\text{Commitments}\\\text{from } S_{t-1}\end{array} + x_t - \begin{array}{c}\text{Predicted}\\\text{Cumulative}\\\text{Expenditures at } S_{t+2}\end{array} \right| \right\}$$

ADP Simulation



Simulate for N iterations:

Step 1 Deterministic: move from pre-decision state to post decision state (PDS)

$$S_t \rightarrow S_t^x = S^{M,x}(S_t, x_t)$$

Step 2 Stochastic: move from post decision state (PDS) to next pre-decision state

$$S_t^x \rightarrow S_{t+1} = S^{M,W}(S_t^x, W_{t+1})$$

Step 3 Repeat process for T months

Q-Learning Output

Comm	Exp	(5.0)	(4.5)	(4.0)	(3.5)	(3.0)	(2.5)	(2.0)	(1.5)	(1.0)	(0.5)	0.0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
0	0	4.593105	3.240517	3.942909	2.506222	2.713528	2.8977	4.342192	3.155769	2.039053	1.238259	4.399158	0	0	0	0	0	0	0	0	0	0
0.5	0	0	4.182829	1.820157	4.442513	2.603491	1.876836	2.584721	3.301983	2.200308	0.316237	3.121326	0.256951	0	0	0	0	0	0	0	0	0
0.5	0.5	0	2.978315	2.19727	2.535189	1.667643	2.360228	1.903759	2.404394	2.90757	4.813437	1.407338	2.783722	0	0	0	0	0	0	0	0	0
1	0	0	0	2.446311	3.014853	1.720142	3.218167	3.87171	4.355842	3.404336	0.235118	2.769361	0.522621	1.420244	0	0	0	0	0	0	0	0
1	0.5	0	0	1.746253	2.231379	4.760294	2.149056	3.146523	2.44335	1.897579	2.022749	0.662734	2.109432	2.264098	0	0	0	0	0	0	0	0
1	1	0	0	1.263345	3.454174	2.079029	1.840341	1.588968	3.092034	2.992489	2.265497	2.444747	1.879282	2.800189	0	0	0	0	0	0	0	0
1.5	0	0	0	0	1.620696	2.055022	4.351505	2.022041	2.411808	2.532145	3.135927	2.239712	0.660561	2.799926	1.726016	0	0	0	0	0	0	0
1.5	0.5	0	0	0	4.314988	1.663595	1.682072	2.363636	1.569646	1.992463	1.826735	0.355203	1.853347	0.830416	2.059368	0	0	0	0	0	0	0
1.5	1	0	0	0	1.492676	1.368906	2.319119	1.669449	3.498979	1.876675	0.972091	1.470205	2.032333	1.870047	2.128677	0	0	0	0	0	0	0
1.5	1.5	0	0	0	2.116797	1.630707	1.308034	1.334396	3.208789	0.467402	2.443047	1.217063	2.504269	3.272062	0.965751	0	0	0	0	0	0	0
2	0	0	0	0	0	2.285307	2.338855	3.35987	4.615637	3.258509	2.994544	3.635513	1.407253	0.317822	3.976046	3.059694	0	0	0	0	0	0
2	0.5	0	0	0	0	2.320516	1.778747	1.80124	1.284532	1.38232	0.34586	0.260081	2.673415	0.02597	0.027527	1.06126	0	0	0	0	0	0
2	1	0	0	0	0	1.343074	2.162401	2.946712	2.586787	1.576827	2.800488	2.608973	0.317857	3.497271	1.518783	2.934368	0	0	0	0	0	0
2	1.5	0	0	0	0	1.331283	1.200556	1.450437	2.303498	2.066899	1.782557	0.547623	3.179604	1.394261	2.071729	1.492124	0	0	0	0	0	0
2	2	0	0	0	0	1.818528	1.049782	1.10884	1.945241	0.544543	0.716931	0.850127	2.117909	1.849248	0.741475	0	0	0	0	0	0	0
2.5	0	0	0	0	0	0	2.920025	3.07924	2.728074	4.397483	3.35777	2.153177	1.795837	3.181114	2.628892	3.102666	2.240804	0	0	0	0	0
2.5	0.5	0	0	0	0	0	3.368114	3.300087	1.701658	2.987257	3.075026	1.794605	2.934653	0.017414	0.017554	3.038411	1.724059	0	0	0	0	0
2.5	1	0	0	0	0	0	2.509793	1.967639	1.288188	1.194157	1.914613	0.038399	0.034709	1.442583	0.853858	1.274971	1.429999	0	0	0	0	0
2.5	1.5	0	0	0	0	0	2.228385	0.776239	1.314933	1.530243	0.930553	1.428295	1.249322	1.895467	2.802327	2.034466	1.826942	0	0	0	0	0
2.5	2	0	0	0	0	0	2.25336	0.933223	1.432997	1.885266	2.243655	1.393963	1.517107	0.692764	2.048173	0.451052	0	0	0	0	0	0
2.5	2.5	0	0	0	0	0	0.949605	0.470348	0.379902	1.831057	1.172398	0.367475	1.54609	1.716537	0.482119	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	3.149052	2.967441	1.644242	4.577967	1.967207	2.197378	1.863188	2.801777	3.490615	2.583073	2.654369	0	0	0	0
3	0.5	0	0	0	0	0	0	3.069655	2.63204	2.5366	1.945799	2.63465	1.841423	1.652242	0.076903	0.072731	1.875528	2.884137	0	0	0	0
3	1	0	0	0	0	0	0	2.677918	3.173489	2.768688	2.934666	2.730004	0.080055	2.317524	0.248332	0.146261	1.108611	1.328518	0	0	0	0
3	1.5	0	0	0	0	0	0	1.191406	2.527674	1.63037	1.942176	1.199113	0.016674	0.016728	0.016793	1.988132	3.116973	1.738701	0	0	0	0
3	2	0	0	0	0	0	0	1.027168	2.841133	2.366059	2.037856	1.154233	2.926934	1.844707	0.016295	1.291345	1.52436	0	0	0	0	0
3	2.5	0	0	0	0	0	0	1.56042	0.750052	1.127166	2.003189	0.445673	0.627824	1.129074	1.501873	1.204346	0	0	0	0	0	0
3	3	0	0	0	0	0	0	0.604309	0.86695	0.701165	0.271282	0.567113	0.411723	0.108271	1.559428	0	0	0	0	0	0	0
3.5	0	0	0	0	0	0	0	0	1.863006	2.355526	2.318502	2.957	2.765366	2.893568	2.559316	3.294894	3.112401	4.398045	4.893306	0	0	0
3.5	0.5	0	0	0	0	0	0	0	3.524906	2.360812	2.028675	1.643429	1.867633	3.698398	1.383996	2.191923	1.053271	2.765881	3.7494	0	0	0
3.5	1	0	0	0	0	0	0	0	0.749138	1.302747	1.521823	1.718323	2.648074	2.367437	1.33171	0.017008	0.282158	1.358296	2.592928	0	0	0
3.5	1.5	0	0	0	0	0	0	0	2.283783	1.330461	1.418729	2.445431	0.016433	0.016516	0.016541	0.016344	3.117331	1.059731	0.969224	0	0	0
3.5	2	0	0	0	0	0	0	0	1.592784	0.599008	0.803637	2.038322	0.865072	0.015969	0.016122	0.958269	2.768699	2.271091	0	0	0	0
3.5	2.5	0	0	0	0	0	0	0	0.328137	1.017807	0.572215	2.166552	1.044566	1.01797	1.261174	1.107751	2.191004	0	0	0	0	0
3.5	3	0	0	0	0	0	0	0	0.491123	0.290359	0.499027	0.541447	0.440648	1.118863	0.351439	0.063733	0	0	0	0	0	0
3.5	3.5	0	0	0	0	0	0	0	0.337134	0.754151	0.042837	0.704714	0.085515	0.208794	0.019131	0	0	0	0	0	0	0

Value Function Output

Comm	Exp	MinV(T)	Counter
0	0	1.238259	1253
0.5	0	0.256951	51351
0.5	0.5	1.407338	562
1	0	0.235118	53506
1	0.5	0.662734	550
1	1	1.263345	395
1.5	0	0.660561	33367
1.5	0.5	0.355203	22892
1.5	1	0.972091	392
1.5	1.5	0.467402	460
2	0	0.317822	5792
2	0.5	0.02597	16853
2	1	0.317857	11234
2	1.5	0.547623	449
2	2	0.544543	401
2.5	0	1.795837	1254
2.5	0.5	0.017414	8730
2.5	1	0.034709	9043
2.5	1.5	0.776239	483
2.5	2	0.451052	395
2.5	2.5	0.367475	389
3	0	1.644242	1305
3	0.5	0.072731	1672
3	1	0.080055	5018
3	1.5	0.016674	8838
3	2	0.016295	1030
3	2.5	0.445673	365
3	3	0.108271	299

Q-Learning and Value Learning Output

Comm	Exp	(5.0)	(4.5)	(4.0)	(3.5)	(3.0)	(2.5)	(2.0)	(1.5)	(1.0)	(0.5)	0.0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
0	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0	0	0	0	0	0	0	0	0	0
0.5	0	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0	0	0	0	0	0	0	0	0
0.5	0.5	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0	0	0	0	0	0	0	0	0
1	0	0	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0	0	0	0	0	0	0	0
1	0.5	0	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0	0	0	0	0	0	0	0
1	1	0	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0	0	0	0	0	0	0	0
1.5	0	0	0	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0	0	0	0	0	0	0
1.5	0.5	0	0	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0	0	0	0	0	0	0
1.5	1	0	0	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0	0	0	0	0	0	0
1.5	1.5	0	0	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0	0	0	0	0	0	0
2	0	0	0	0	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0	0	0	0	0	0
2	0.5	0	0	0	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0	0	0	0	0	0
2	1	0	0	0	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0	0	0	0	0	0
2	1.5	0	0	0	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0	0	0	0	0	0
2	2	0	0	0	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0	0	0	0	0	0	0
2.5	0	0	0	0	0	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0	0	0	0	0
2.5	0.5	0	0	0	0	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0	0	0	0	0
2.5	1	0	0	0	0	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0	0	0	0	0
2.5	1.5	0	0	0	0	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0	0	0	0	0
2.5	2	0	0	0	0	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0	0	0	0	0	0
2.5	2.5	0	0	0	0	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0	0	0	0
3	0.5	0	0	0	0	0	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0	0	0	0
3	1	0	0	0	0	0	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0	0	0	0
3	1.5	0	0	0	0	0	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0	0	0	0
3	2	0	0	0	0	0	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0	0	0	0	0
3	2.5	0	0	0	0	0	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0	0	0	0	0	0
3	3	0	0	0	0	0	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0	0	0	0	0	0	0
3.5	0	0	0	0	0	0	0	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0	0	0
3.5	0.5	0	0	0	0	0	0	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0	0	0
3.5	1	0	0	0	0	0	0	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0	0	0
3.5	1.5	0	0	0	0	0	0	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0	0	0
3.5	2	0	0	0	0	0	0	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0	0	0	0
3.5	2.5	0	0	0	0	0	0	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0	0	0	0	0
3.5	3	0	0	0	0	0	0	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0	0	0	0	0	0
3.5	3.5	0	0	0	0	0	0	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0	0
4	0.5	0	0	0	0	0	0	0	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0	0
4	1	0	0	0	0	0	0	0	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0	0
4	1.5	0	0	0	0	0	0	0	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0	0
4	2	0	0	0	0	0	0	0	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0	0	0
4	2.5	0	0	0	0	0	0	0	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0	0	0	0
4	3	0	0	0	0	0	0	0	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0	0	0	0	0
4	3.5	0	0	0	0	0	0	0	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0	0	0	0	0	0
4	4	0	0	0	0	0	0	0	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0	0	0	0	0	0	0
4.5	0	0	0	0	0	0	0	0	0	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0
4.5	0.5	0	0	0	0	0	0	0	0	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0
4.5	1	0	0	0	0	0	0	0	0	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0
4.5	1.5	0	0	0	0	0	0	0	0	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0
4.5	2	0	0	0	0	0	0	0	0	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0	0
4.5	2.5	0	0	0	0	0	0	0	0	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0	0	0
4.5	3	0	0	0	0	0	0	0	0	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0	0	0	0
4.5	3.5	0	0	0	0	0	0	0	0	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0	0	0	0	0
4.5	4	0	0	0	0	0	0	0	0	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0	0	0	0	0	0
4.5	4.5	0	0	0	0	0	0	0	0	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
5	0.5	0	0	0	0	0	0	0	0	0	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
5	1	0	0	0	0	0	0	0	0	0	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
5	1.5	0	0	0	0	0	0	0	0	0	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
5	2	0	0	0	0	0	0	0	0	0	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0
5	2.5	0	0	0	0	0	0	0	0	0	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0	0
5	3	0	0	0	0	0	0	0	0	0	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0	0	0
[illegible]	[illegible]	0	0	0	0	0	0	0	0	0	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0	0	0	0
[illegible]	[illegible]	0	0	0	0	0	0	0	0	0	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0	0	0	0	0
[illegible]	[illegible]	0	0	0	0	0	0	0	0	0	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0	0	0	0	0	0
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

Comm	Exp	MinV(T)	Counter
0	0	11.92188	1848
0.5	0	13.49567	1882
0.5	0.5	4.796836	852
1	0	10.17801	1938
1	0.5	8.357031	856
1	1	4.91157	822
1.5	0	11.39166	1881
1.5	0.5	2.028904	66606
1.5	1	5.716428	791
1.5	1.5	3.166108	720
2	0	3.287661	215534
2	0.5	5.055065	833
2	1	5.010837	749
2	1.5	4.428067	761
2	2	4.220616	663
2.5	0	8.469443	5903
2.5	0.5	6.279228	878
2.5	1	5.930043	780
2.5	1.5	2.986717	765
2.5	2	3.263341	653
2.5	2.5	2.368588	486
3	0	10.09113	1812
3	0.5	6.506079	837
3	1	5.70113	769
3	1.5	4.108784	706
3	2	3.719395	596
3	2.5	1.595096	524
3	3	1.36571	324
3.5	0	10.96392	1946
3.5	0.5	6.01121	856
3.5	1	1.01322	53573
3.5	1.5	8.119015	736
3.5	2	2.693575	674
3.5	2.5	1.808178	552
3.5	3	0.812264	364
3.5	3.5	0.076253	167
4	0	13.91128	1837
4	0.5	1.042773	26004
4	1	6.058729	796
4	1.5	0.697462	31005
4	2	3.607565	722
4	2.5	1.89915	523
4	3	1.074464	339
4	3.5	0.459107	199
4	4	0.146779	80
4.5	0	12.58923	1939
4.5	0.5	6.037655	855
4.5	1	5.234122	776
4.5	1.5	3.129337	746
4.5	2	0.402692	50309
4.5	2.5	0.083986	32819
4.5	3	2.00322	317
4.5	3.5	0.744718	176
4.5	4	0.218768	66
4.5	4.5	0.013632	23
5	0	1.273092	5046
5	0.5	7.698101	768
5	1	0.67575	22946
5	1.5	0.62733	[illegible]
5	2	3.118608	[illegible]

Step-Size Properties

- **Use a smoothing algorithm to generate an estimate for $\bar{V}_t^n(S_t^n)$**

$$\bar{V}_t^n(S_t^n) = (1 - \propto_{n-1})\bar{V}_t^{n-1}(S_t^n) + \propto_{n-1} (\hat{v}_t^n)$$

- **$\propto_{n-1}$ is referred to as the step-size, alpha-decay, or learning rate**
- **Alpha-decay properties for convergence:**

$$\alpha_{n-1} \geq 0, \qquad n = 1,2, \ldots$$

$$\sum_{n=1}^{\infty} \alpha_{n-1} = \infty$$

$$\sum_{n=1}^{\infty} (\alpha_{n-1})^2 < \infty$$

Step-Size Properties

$$\bar{\theta}^n = (1 - \alpha_{n-1})\bar{\theta}^{n-1} + \alpha_{n-1}\hat{\theta}^n$$

- **Step-Size Choice**
  - Convergence conditions ensure that step-size will decay/decrease during each successive iteration
  - Each iteration slightly shifts the weight from the observation $\hat{\theta}^n$ to the mean/signal $\bar{\theta}^{n-1}$
  - Alpha-Decay too slow run the risk of algorithm stalling out
  - Alpha-Decay too fast run the risk of apparent convergence (no learning)
  - Used Adapted Determinstic Harmonic Alpha-Decay as recommended by Darken et al. (1992) / Gosavi (2003)
    - Only requires tuning a single term β
    - Alpha term remains high during earlier iterations giving algorithm time to learn before decreasing rapidly

$$\alpha_n = \frac{\alpha_{n-1}}{1+\frac{n^2}{\beta+n}}$$

Test Case Summaries: Q-Learning

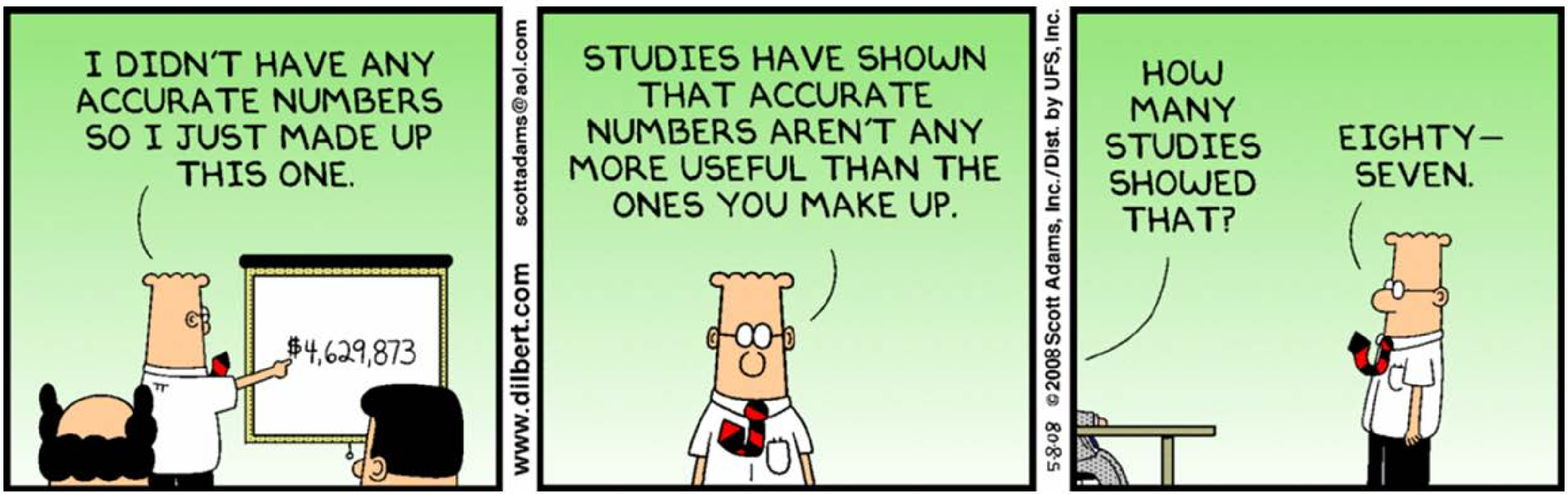
#	Name:	Model Type	# of Projects:	# of Mths:	$M Total Budget:	Exp Ite:	MSE:	Alpha-Decay	# of States	Run Time	Parameters	Proj. #1	Proj. #2	Proj. #3	Proj. #4
1	TestCase#1 Trial#1	Q-Learning	2	12	7.000	Exp: 5,000 Lrn: 50,000	Last 10,000 observations all under 0.6	Exp: 0.8 to 0.6 Lrn: 0.6 to 0.1	435	4hrs 34Min	Budget	5.000	2.000		
											Alloc. Param.	0.500	0.250		
											+/- Factor	1.000	0.750		
	Notes: Intel® Xeon® 2.33GHz, 3.00 GB RAM Q-Matrix: visited acceptable state-spaces														
2	TestCase#1 Trial#2	Q-Learning	2	12	7.000	Exp: 5,000 Lrn: 75,000	Last 10,000 observations all under 0.15	Exp: 0.8 to 0.6 Lrn: 0.6 to 0.1	435	Did Not Record	Budget	5.000	2.000		
											Alloc. Param.	0.500	0.250		
											+/- Factor	1.000	0.750		
	Notes: Intel® Xeon® 2.33GHz, 3.00 GB RAM Q-Matrix: visited acceptable state-spaces; noticeable decrease in MSE Statistic from TC#1Trial #1														
3	TestCase#1 Trial#3	Q-Learning	2	12	7.000	Exp: 5,000 Lrn: 100,000	Last 10,000 observations all under 0.3	Exp: 0.9 to 0.6 Lrn: 0.6 to 0.1	435	Did Not Record	Budget	5.000	2.000		
											Alloc. Param.	0.500	0.250		
											+/- Factor	1.000	0.750		
	Notes: Intel® Xeon® 2.33GHz, 3.00 GB RAM Q-Matrix: visited acceptable state-spaces; slight increases in MSE from TC#1Trial#2														
4	TestCase#2 Trial#1	Q-Learning	3	12	22.000	Exp: 5,000 Lrn: 50,000	Last 10,000 observations between 2 and 25	Exp: 0.8 to 0.6 Lrn: 0.6 to 0.1	4,005	23 hrs 11Min	Budget	5.000	2.000	15.000	
											Alloc. Param.	0.500	0.250	1.000	
											+/- Factor	1.000	0.750	2.000	
	Notes: Intel® Xeon® 2.33GHz, 3.00 GB RAM Q-Matrix: visited acceptable state-spaces; Q-Matrix: some sparsity: a number viable state-spaces not visited														
5	TestCase#3 Trial#1	Q-Learning	4	24	68.000	Exp: 5,000 Lrn: 50,000	Last 10,000 observations most under 5.0 some between 5.0 & 10	Exp: 0.8 to 0.6 Lrn: 0.6 to 0.1	2,415	1 day 21hrs 6Min	Budget	40.000	10.000	6.000	12.000
											Alloc. Param.	2.000	1.000	1.000	2.000
											+/- Factor	6.000	1.00	2.000	4.000
	Notes: Intel® Xeon® 2.33GHz, 3.00 GB RAM Q-Matrix: visited acceptable state-spaces: Q-Matrix: some sparsity: a number of viable state-spaces not visited compared to TC#2Trial#1: fewer state-spaces, doubled number of months -- run-time nearly doubled														

Test Case Summaries: V-Learning

#	Name:	Model Type	# of Projects:	# of Mths:	$M Total Budget:	Exp Ite:	MSE:	Alpha-Decay	# of States	Run Time	Parameters	Proj. #1	Proj. #2	Proj. #3	Proj. #4
6	**TestCase#1 Trial#1** Notes: Intel® Core® 2.40GHz, 6.00 GB RAM different platfrom and V-Learning appear to improve run-time	Value Learning	2	12	7.000	Exp: 5,000 Lrn: 50,000	Last 10,000 observations all under 0.6	Exp: 0.8 to 0.6 Lrn: 0.6 to 0.1	435	2hrs 15Min	**Budget**	5.000	2.000		
											Alloc. Param.	0.500	0.250		
											+/- Factor	1.000	0.750		
7	**TestCase#1 Trial#2** Notes: Intel® Core® 2.70GHz, 8.00 GB RAM number of months appears to impact run-times heavily	Value Learning	2	16	9.500	Exp: 5,000 Lrn: 50,000	750,000 MSE data points	Exp: 0.8 to 0.6 Lrn: 0.6 to 0.1	780	3hrs 53Min	**Budget**	7.000	2.500		
											Alloc. Param.	0.500	0.250		
											+/- Factor	0.500	0.250		
8	**TestCase#2 Trial#1** Notes: Intel® Core® 2.70GHz, 8.00 GB RAM V-Learning apppears to improve run-time	Value Learning	3	12	22.000	Exp: 5,000 Lrn: 50,000	Last 10,000 observations between 2 and 25	Exp: 0.8 to 0.6 Lrn: 0.6 to 0.1	4,005	11hrs 18Min	**Budget**	5.000	2.000	15.000	
											Alloc. Param.	0.500	0.250	1.000	
											+/- Factor	1.000	0.750	2.000	
9	**TestCase#2 Trial#2** Notes: Intel® Xeon® 2.33GHz, 3.00 GB RAM slight improvement on MSE from TestCase#2Trial#1	Value Learning	3	12	22.000	Exp: 5,000 Lrn: 75,000	Last 10,000 observations between 1 and 20	Exp: 0.8 to 0.6 Lrn: 0.6 to 0.1	4,005	18hrs 10Min	**Budget**	5.000	2.000	15.000	
											Alloc. Param.	0.500	0.250	1.000	
											+/- Factor	1.000	0.750	2.000	
10	**TestCase#2 Trial#3** Notes: Intel® Xeon® 2.33GHz, 3.00 GB RAM at start of learning, MSE statistics higher than TestCase#2Trial#2	Value Learning	3	12	22.000	Exp: 10,000 Lrn: 100,000	Last 10,000 observations between 1 and 20	Exp: 0.9 to 0.6 Lrn: 0.6 to 0.1	4,005	1 day 7hrs 10Min	**Budget**	5.000	2.000	15.000	
											Alloc. Param.	0.500	0.250	1.000	
											+/- Factor	1.000	0.750	2.000	

Test Case Summaries: V-Learning (Cont.)

#	Name:	Model Type	# of Projects:	# of Mths:	$M Total Budget:	Exp Ite:	MSE:	Alpha-Decay	# of States	Run Time	Parameters	Proj. #1	Proj. #2	Proj. #3	Proj. #4
11	**TestCase#3 Trial#1**	Value Learning	3	12	22.000	Exp: 5,000 Lrn: 50,000	Last 10,000 observations between 2 and 25	Exp: 0.8 to 0.6 Lrn: 0.6 to 0.1	4,005	11hrs 57Min	Budget	5.000	2.000	15.000	
											Alloc. Param.	0.500	0.250	1.000	
											+/- Factor	1.000	0.750	2.000	
	Notes: Intel® Xeon® 2.33GHz, 3.00 GB RAM slight improvement on MSE from TestCase#2Trial#1 re-phasing initial commitment/expenditure planning appears to smooth ADP strategy compated with TestCae#2														
12	**TestCase#3 Trial#2**	Value Learning	3	12	22.000	Exp: 5,000 Lrn: 75,000	Last 10,000 observations between 1 and 20	Exp: 0.8 to 0.6 Lrn: 0.6 to 0.1	4,005	17hrs 31Min	Budget	5.000	2.000	15.000	
											Alloc. Param.	0.500	0.250	1.000	
											+/- Factor	1.000	0.750	2.000	
	Notes: Intel® Xeon® 2.33GHz, 3.00 GB RAM slight improvement on MSE from TestCase#3Trial#1														
13	**TestCase#3 Trial#3**	Value Learning	3	12	22.000	Exp: 5,000 Lrn: 100,000	Last 10,000 observations between 1 and 20	Exp: 0.8 to 0.6 Lrn: 0.6 to 0.1	4,005	23hrs 00Min	Budget	5.000	2.000	15.000	
											Alloc. Param.	0.500	0.250	1.000	
											+/- Factor	1.000	0.750	2.000	
	Notes: Intel® Xeon® 2.33GHz, 3.00 GB RAM no noticeable improvement on MSE from TestCase#3Trial#2														
14	**TestCase#3 Trial#4**	Value Learning	3	12	22.000	Exp: 5,000 Lrn: 150,000	Last 10,000 observations between 1 and 15	Exp: 0.8 to 0.6 Lrn: 0.6 to 0.1	4,005	1 day 10hrs 06Min	Budget	5.000	2.000	15.000	
											Alloc. Param.	0.500	0.250	1.000	
											+/- Factor	1.000	0.750	2.000	
	Notes: Intel® Xeon® 2.33GHz, 3.00 GB RAM slight improvement on MSE from TestCase#3Trial#3														
15	**TestCase#3 Trial#5**	Value Learning	3	12	22.000	Exp: 5,000 Lrn: 175,000	Last 10,000 observations between 1 and 17	Exp: 1.0 to 0.8 Lrn: 0.8 to 0.1	4,005	1 day 16hrs 27Min	Budget	5.000	2.000	15.000	
											Alloc. Param.	0.500	0.250	1.000	
											+/- Factor	1.000	0.750	2.000	
	Notes: Intel® Xeon® 2.33GHz, 3.00 GB RAM Used an alternative deterministic alpha-decay parameter Powell (2007) no noticeable improvement on MSE data compared to Trials#1-#4														
16	**TestCase#3 Trial#6**	Value Learning	3	12	22.000	Exp: 5,000 Lrn: 100,000	Last 10,000 observations between 2 and 6	Exp: 0.8 to 0.6 Lrn: 0.6 to ~0.0	4,005	15hrs 41Min	Budget	5.000	2.000	15.000	
											Alloc. Param.	0.500	0.250	1.000	
											+/- Factor	1.000	0.750	2.000	
	Notes: Intel® Core® 2.70GHz, 8.00 GB RAM deliberately timed-out alpha-decay; alpha-decay reached 0.1 at the 75,000 iteration of learning variance much tighter during final iterations - stabilized MSE statistic														
17	**TestCase#4 Trial#1**	Value Learning	4	12	44.000	Exp: 5,000 Lrn: 50,000	Last 10,000 observations between 3 and 40	Exp: 0.8 to 0.6 Lrn: 0.6 to 0.1	4,005	16hrs 30Mins	Budget	12.000	7.000	15.000	10.000
											Alloc. Param.	1.000	0.500	1.000	0.500
											+/- Factor	2.000	1.500	2.000	2.000
	Notes: Intel® Core® 2.70GHz, 8.00 GB RAM learnt phase results consistent w/TestCase#2 & #3														


I DIDN'T HAVE ANY ACCURATE NUMBERS SO I JUST MADE UP THIS ONE.
$4,629,873
www.dilbert.com scottadams@aol.com
STUDIES HAVE SHOWN THAT ACCURATE NUMBERS AREN'T ANY MORE USEFUL THAN THE ONES YOU MAKE UP.
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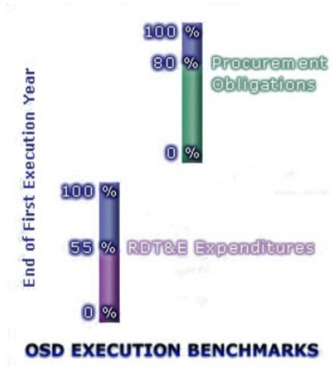
HOW MANY STUDIES SHOWED THAT?
EIGHTY-SEVEN.

Color of Money – Spending Benchmarks

Financial ManagementOSD Execution Benchmarks

acqnotes.com/acqnote/careerfields/osd-execution-benchmarks



OSD analysts expect to see at least 80% of Procurement funds obligated by the end of the first year of execution of those funds. For RDT&E appropriations, they expect expenditures of at least 55% of the amount appropriated by the end of the first year of execution. [1]

These benchmarks are derived from historical information in the official accounting records. Thus, common delays inherent in the accounting process are automatically accounted for in these benchmarks, and will not generally be accepted as an explanation of why the program appears to be lagging in its funds execution.

However, if the program can present credible evidence that unusual delays or errors have occurred in the processing of its obligations or expenditures (such as postings to the wrong fund cites), this will often be accepted and considered by the analyst in making recommendations for the program.

Programs that over several years consistently execute their funds at rates below the benchmarks will find it very difficult to defend themselves against funding adjustments.

Appropriation Category	First Year Available		Cumulative for Second Year		Cumulative for Third Year	
	Obligation	Expenditures	Obligation	Expenditure	Obligation	Expenditure
O&M	100%	75%	100%	100%	100%	100%
RDT&E	90%	55%	100%	90%	100%	100%
Procurement	80%	N/A	90%	N/A	100%	N/A
Initial Spares	92%	N/A	96%	N/A	100%	N/A
Adv Proc	100%	N/A	100%	N/A	100%	N/A

AcqLinks and References:

[1] Website: DAU – Program Execution: Evaluating Budget Execution

1/2

Color of Money – Spending Benchmarks

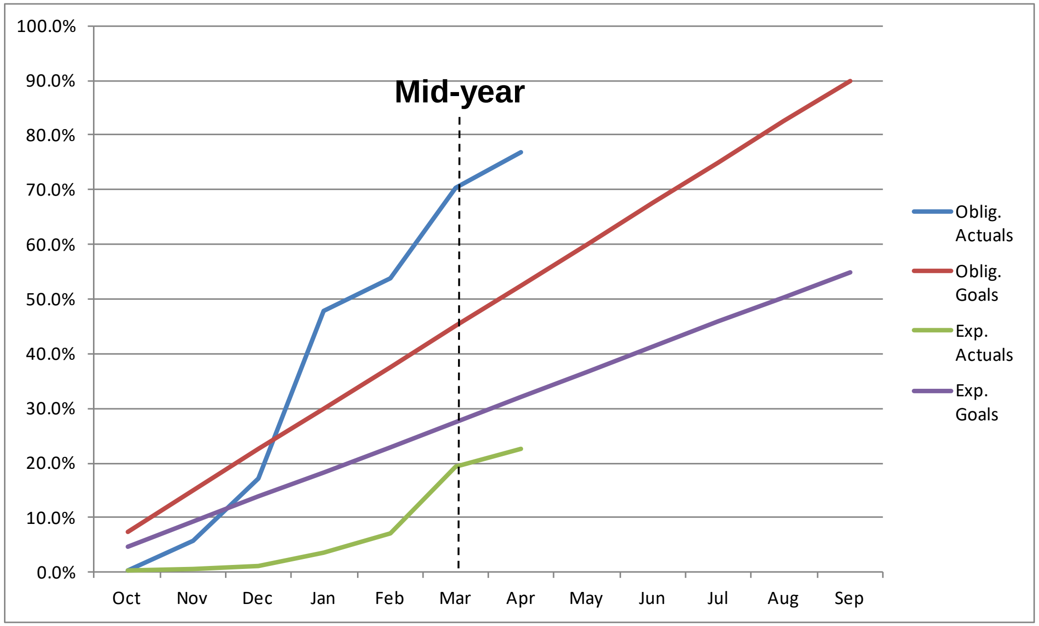
		RDT&E		Procurement		O&M		MILCON	
	Month	Obl.	Exp.	Obl.	Exp.	Obl.	Exp.	Obl.	Exp.
First Year of Availablity	Oct	7.50%	4.60%	6.70%	N/A	8.30%	6.30%	5.40%	1.20%
	Nov	15.00%	9.20%	13.30%	N/A	16.70%	12.50%	10.80%	2.30%
	Dec	22.50%	13.80%	20.00%	N/A	25.00%	18.80%	16.30%	3.50%
	Jan	30.00%	18.30%	26.70%	N/A	33.30%	25.00%	21.70%	4.70%
	Feb	37.50%	22.90%	33.30%	N/A	41.70%	31.30%	27.10%	5.80%
	Mar	45.00%	27.50%	40.00%	N/A	50.00%	37.50%	32.50%	7.00%
	Apr	52.50%	32.10%	46.70%	N/A	58.30%	43.80%	37.90%	8.20%
	May	60.00%	36.70%	53.30%	N/A	66.70%	50.00%	43.30%	9.30%
	Jun	67.50%	41.30%	60.00%	N/A	75.00%	56.30%	48.80%	10.50%
	Jul	75.00%	45.80%	66.70%	N/A	83.30%	62.50%	54.20%	11.70%
	Aug	82.50%	50.40%	73.30%	N/A	91.70%	68.80%	59.60%	12.80%
	Sep	90.00%	55.00%	80.00%	N/A	100.00%	75.00%	65.00%	14.00%
Second Year of Availablity	Oct	90.80%	57.90%	80.80%	N/A	100.00%	77.10%	67.10%	18.10%
	Nov	91.70%	60.80%	81.70%	N/A	100.00%	79.20%	69.20%	22.20%
	Dec	92.50%	63.80%	82.50%	N/A	100.00%	81.30%	71.30%	26.30%
	Jan	93.30%	66.70%	83.30%	N/A	100.00%	83.30%	73.30%	30.30%
	Feb	94.20%	69.60%	84.20%	N/A	100.00%	85.40%	75.40%	34.40%
	Mar	95.00%	72.50%	85.00%	N/A	100.00%	87.50%	77.50%	38.50%
	Apr	95.80%	75.40%	85.80%	N/A	100.00%	89.60%	79.60%	42.60%
	May	96.70%	78.30%	86.70%	N/A	100.00%	91.70%	81.70%	46.70%
	Jun	97.50%	81.30%	87.50%	N/A	100.00%	93.80%	83.80%	50.80%
	Jul	98.30%	84.20%	88.30%	N/A	100.00%	95.80%	85.80%	54.80%
	Aug	99.20%	87.10%	89.20%	N/A	100.00%	97.90%	87.90%	58.90%
	Sep	100.00%	90.00%	90.00%	N/A	100.00%	100.00%	90.00%	63.00%
Third Year of Availablity	Oct	100.00%	90.80%	90.80%	N/A	100.00%	100.00%	90.40%	65.50%
	Nov	100.00%	91.70%	91.70%	N/A	100.00%	100.00%	90.80%	68.10%
	Dec	100.00%	92.50%	92.50%	N/A	100.00%	100.00%	91.30%	70.60%
	Jan	100.00%	93.30%	93.30%	N/A	100.00%	100.00%	91.70%	73.20%
	Feb	100.00%	94.20%	94.20%	N/A	100.00%	100.00%	92.10%	75.70%
	Mar	100.00%	95.00%	95.00%	N/A	100.00%	100.00%	92.50%	78.30%
	Apr	100.00%	95.80%	95.80%	N/A	100.00%	100.00%	92.90%	80.80%
	May	100.00%	96.70%	96.70%	N/A	100.00%	100.00%	93.30%	83.30%
	Jun	100.00%	97.50%	97.50%	N/A	100.00%	100.00%	93.80%	85.90%
	Jul	100.00%	98.30%	98.30%	N/A	100.00%	100.00%	94.20%	88.40%
	Aug	100.00%	99.20%	99.20%	N/A	100.00%	100.00%	94.60%	91.00%
	Sep	100.00%	100.00%	100.00%	N/A	100.00%	100.00%	95.00%	93.50%

Current Method

		RDT&E	
	Month	Obl.	Exp.
First Year of Availabilty	Oct	7.50%	4.60%
	Nov	15.00%	9.20%
	Dec	22.50%	13.80%
	Jan	30.00%	18.30%
	Feb	37.50%	22.90%
	Mar	45.00%	27.50%
	Apr	52.50%	32.10%
	May	60.00%	36.70%
	Jun	67.50%	41.30%
	Jul	75.00%	45.80%
	Aug	82.50%	50.40%
	Sep	90.00%	55.00%
Second Year of Availabilty	Oct	90.80%	57.90%
	Nov	91.70%	60.80%
	Dec	92.50%	63.80%
	Jan	93.30%	66.70%
	Feb	94.20%	69.60%
	Mar	95.00%	72.50%
	Apr	95.80%	75.40%
	May	96.70%	78.30%
	Jun	97.50%	81.30%
	Jul	98.30%	84.20%
	Aug	99.20%	87.10%
	Sep	100.00%	90.00%
Third Year of Availabilty	Oct	100.00%	90.80%
	Nov	100.00%	91.70%
	Dec	100.00%	92.50%
	Jan	100.00%	93.30%
	Feb	100.00%	94.20%
	Mar	100.00%	95.00%
	Apr	100.00%	95.80%
	May	100.00%	96.70%
	Jun	100.00%	97.50%
	Jul	100.00%	98.30%
	Aug	100.00%	99.20%
	Sep	100.00%	100.00%

Mid-Year Execution Goal:
45.00 % of RDT&E Funds Obligated
27.50 % of RDT&E Funds Expended

End-Year Execution Goal:
90.00 % of RDT&E Funds Obligated
55.00 % of RDT&E Funds Expended



End-year expenditure goal is 55%, this leaves 45% still unexpended

Future Research (ADP)

- **Drawback with ADP:**
  - **Scalability issues when state space explodes**
    - **Leads to Computational Complexity**

- **Potential solution**
  - **Value Function Approximation techniques**
    - Aggregation of the outcome space
    - Function fitting / basis functions

$$\overline{V}(S^x) = \sum_{f \in F} \theta_f \phi_f(S)$$

Why ADP?

- **ADP is designed to solve sequential decision making problems in environments of uncertainty**

- **Learns by interacting with environment as system evolves**
  - Calculations only require data from two consecutive time periods

- **ADP possesses following computational advantages**
  - Does not require solving a system of linear equations
  - Does not attempt to solve all time period constraints simultaneously
  - Easily incorporates uncertainty
  - Easily incorporates non-linear cost/reward function