NPS-AM-11-C8P21R01-071

EXCERPT FROM THE
PROCEEDINGS

OF THE

EIGHTH ANNUAL ACQUISITION RESEARCH SYMPOSIUM
THURSDAY SESSIONS
VOLUME II

Optimal Cost Avoidance Investment and Pricing Strategies for Performance-Based Post-Production Service Contracts

David Nowicki, Jose Ramirez-Marquez, and Ilona Murynets, Stevens Institute of Technology, and Wesley Randall, University of North Texas

Published: 30 April 2011

The research presented at the symposium was supported by the Acquisition Chair of the Graduate School of Business & Public Policy at the Naval Postgraduate School.

To request Defense Acquisition Research or to become a research sponsor, please contact:

NPS Acquisition Research Program
Attn: James B. Greene, RADM, USN, (Ret.)
Acquisition Chair
Graduate School of Business and Public Policy
Naval Postgraduate School
555 Dyer Road, Room 332
Monterey, CA 93943-5103
Tel: (831) 656-2092
Fax: (831) 656-2253
E-mail: jbgreene@nps.edu

Copies of the Acquisition Sponsored Research Reports may be printed from our website www.acquisitionresearch.net

Preface & Acknowledgements

During his internship with the Graduate School of Business & Public Policy in June 2010, U.S. Air Force Academy Cadet Chase Lane surveyed the activities of the Naval Postgraduate School's Acquisition Research Program in its first seven years. The sheer volume of research products—almost 600 published papers (e.g., technical reports, journal articles, theses)—indicates the extent to which the depth and breadth of acquisition research has increased during these years. Over 300 authors contributed to these works, which means that the pool of those who have had significant intellectual engagement with acquisition issues has increased substantially. The broad range of research topics includes acquisition reform, defense industry, fielding, contracting, interoperability, organizational behavior, risk management, cost estimating, and many others. Approaches range from conceptual and exploratory studies to develop propositions about various aspects of acquisition, to applied and statistical analyses to test specific hypotheses. Methodologies include case studies, modeling, surveys, and experiments. On the whole, such findings make us both grateful for the ARP's progress to date, and hopeful that this progress in research will lead to substantive improvements in the DoD's acquisition outcomes.

As pragmatists, we of course recognize that such change can only occur to the extent that the potential knowledge wrapped up in these products is put to use and tested to determine its value. We take seriously the pernicious effects of the so-called "theory–practice" gap, which would separate the acquisition scholar from the acquisition practitioner, and relegate the scholar's work to mere academic "shelfware." Some design features of our program that we believe help avoid these effects include the following: connecting researchers with practitioners on specific projects; requiring researchers to brief sponsors on project findings as a condition of funding award; "pushing" potentially high-impact research reports (e.g., via overnight shipping) to selected practitioners and policy-makers; and most notably, sponsoring this symposium, which we craft intentionally as an opportunity for fruitful, lasting connections between scholars and practitioners.

A former Defense Acquisition Executive, responding to a comment that academic research was not generally useful in acquisition practice, opined, "That's not their [the academics'] problem—it's ours [the practitioners']. They can only perform research; it's up to us to use it." While we certainly agree with this sentiment, we also recognize that any research, however theoretical, must point to some termination in action; academics have a responsibility to make their work intelligible to practitioners. Thus we continue to seek projects that both comport with solid standards of scholarship, and address relevant acquisition issues. These years of experience have shown us the difficulty in attempting to balance these two objectives, but we are convinced that the attempt is absolutely essential if any real improvement is to be realized.

We gratefully acknowledge the ongoing support and leadership of our sponsors, whose foresight and vision have assured the continuing success of the Acquisition Research Program:

- Office of the Under Secretary of Defense (Acquisition, Technology & Logistics)
- Program Executive Officer SHIPS
- Commander, Naval Sea Systems Command
- Army Contracting Command, U.S. Army Materiel Command
- Program Manager, Airborne, Maritime and Fixed Station Joint Tactical Radio System

- Program Executive Officer Integrated Warfare Systems
- Office of the Assistant Secretary of the Air Force (Acquisition)
- Office of the Assistant Secretary of the Army (Acquisition, Logistics, & Technology)
- Deputy Assistant Secretary of the Navy (Acquisition & Logistics Management)
- Director, Strategic Systems Programs Office
- Deputy Director, Acquisition Career Management, US Army
- Defense Business Systems Acquisition Executive, Business Transformation Agency
- Office of Procurement and Assistance Management Headquarters, Department of Energy

We also thank the Naval Postgraduate School Foundation and acknowledge its generous contributions in support of this Symposium.

James B. Greene, Jr.
Rear Admiral, U.S. Navy (Ret.)

Keith F. Snider, PhD
Associate Professor

Panel 21 – Innovative Mechanisms for Improved Acquisition

Thursday, May 12, 2011	
1:45 p.m. – 3:15 p.m.	**Chair: Dr. Fred Thompson,** Professor, Atkinson Graduate School of Management, Willamette University ***Optimal Cost Avoidance Investment and Pricing Strategies for Performance-Based Post-Production Service Contracts*** David Nowicki, Jose Ramirez-Marquez, and Ilona Murynets, Stevens Institute of Technology, and Wesley Randall, University of North Texas ***Prediction Markets as an Information Aggregation Tool for Effective Project Management in Defense Acquisition Projects*** Ricardo Valerdi, Massachusetts Institute of Technology, and Matthew Potoski, Iowa State University ***Game Theoretic Real Option Approach of the Procurement of Department of Defense: Competition or Collaboration*** Marc Rabaey, Belgian MoD, University of Hasselt

Fred Thompson—Grace and Elmer Goudy Professor of Public Management and Policy Analysis at the Atkinson Graduate School of Management, Willamette University. Dr. Thompson is a specialist in the field of tax policy and regulation.

Dr. Thompson is co-editor of the *Handbook of Public Finance*. He was the founding editor of the *International Public Management Journal* and is currently associate editor of the *Journal of Comparative Policy Analysis*. He has been published in numerous scholarly journals, including the *American Political Science Review*, *Public Administration Review*, *Public Choice*, and *Journal of Economic Behavior and Organization*.

In 2000, Dr. Thompson received the Distinguished Research Award of the National Association of Schools of Public Affairs and Administration and the American Society for Public Administration. In 2005 he received the Aaron B. Wildavsky Award for Outstanding Lifetime Scholarly Achievement in the field of public budgeting and financial management of the Association for Budgeting and Financial Management. In 2006 he served on the United Nations Development Program's Blue Ribbon Commission on Macedonia.

Dr. Thompson earned his Bachelor of Arts in Economics and History from Pomona College and his PhD from the Center for Politics and Economics, Claremont Graduate University.

Optimal Cost Avoidance Investment and Pricing Strategies for Performance-Based Post-Production Service Contracts

David Nowicki—Associated Professor, School of Systems and Enterprises, Stevens Institute of Technology (PhD and BS, University of Wisconsin-Madison, MS Virginia Tech). Dr. Nowicki's research focus is on the economic examination of the intersection between system designs and their supporting sustainment infrastructure. His research interests include performance-based logistics modeling, supply chain management, multi-asset optimization, reliability theory, and inventory optimization. He has been published in top-tier academic journals, conducted seminars, and presented at conferences in the United States, Europe, and APAC. Dr. Nowicki has over 15 years of industry experience. [dnowicki@stevens.edu]

Jose Ramirez-Marquez—Associate Professor, School of Systems and Enterprises, Stevens Institute of Technology. A former Fulbright Scholar, Dr. Ramirez-Marquez holds degrees from Rutgers University in Industrial Engineering (PhD and MS) and Statistics (MS) and from Universidad Nacional Autonoma de Mexico in Actuarial Science. His research efforts focus on developing mathematical models for the analysis, computation, and optimization of system performance with special interest in complex network operational effectiveness and system resilience. Dr. Ramirez-Marquez has conducted funded research for both private industry and government and has over 70 refereed publications. [jmarquez@stevens.edu]

Wesley Randall—Assistant Professor, Logistics, University of North Texas (PhD, University of North Texas, MMS Marine Command and Staff College, MPA Valdosta State University, BS United States Air Force Academy). Dr. Randall is a retired USAF Officer. He served as an operational logistician and commander employing the F-16, A-10, and F-117 in peacetime and war; as an acquisition officer supporting the F-22 and the F-16; and as a Joint Staff Officer for NATO's fleet of AWACS. Dr. Randall's research interests include the econometrics associated with outcome based sustainment strategies. Dr. Randall teaches supply chain decision-making and air transportation management. He has on-going research funding from both commercial firms and government agencies. His research has been accepted in top-tier aerospace, logistics, and supply chain business journals. [wesley.randall@unt.edu]

Ilona Murynets—Scientist, Chief Security Office, AT&T. Dr. Murynets recently completed a postdoctoral research appointment at Stevens Institute of Technology. She obtained her PhD in Systems Engineering at the School of Systems and Enterprises, Stevens Institute of Technology, where her dissertation received an Outstanding Dissertation Award. She holds a BS degree in Mathematics and an MS degree in Statistics and Financial and Actuarial Mathematics from Kiev National Taras Shevchenko University, Ukraine. Dr. Murynets' research is in the area of data mining, optimization, and statistical analysis in application to service pricing, malware propagation, spam detection, and mobile and network security. [imurynet@stevens.edu]

Abstract

Performance-based contracting (PBC) is altering the fundamental relationship between buyers and suppliers engaged in the support of capital-intensive systems such as high-speed rail, defense, and power generation. This relationship is shifting from a traditional transactional-based (return on sales) business approach to a collaborative, performance-based (return on investment) multi-year contractual model. With PBC, the supplier is compensated for system performance rather than for each maintenance, repair, and overhaul (MRO) transaction. PBC success lies in the incentive structure. Under PBC supplier profits, system performance and operator costs are improved when smart investment decisions are made that trade year after year MRO costs for upfront investments that reduce total cost of ownership.

This paper develops a decision-theoretic model that determines the optimal contract length and optimal investment and pricing strategies for performance-based, post-production service contracts that simultaneously maximizes the profit to the supplier while satisfying the customer's needs. The model accounts for reliability as a function investment, the average and variance of the cost to perform maintenance tasks, and for customers' willingness to pay for a contract depending on its length. Numerical examples illustrate how optimal strategies depend on potential market size, expected cost per failure, and on other parameters of the model.

Introduction

There is a noticeable paradigm shift in the contractual relationship between suppliers and buyers of post-production support service. Traditionally buyers and suppliers of post-production support for high capital intensive systems (e.g., high speed rail, defense systems, and power distribution systems) have tended to adopt a transactional relationship (Sols, Nowicki, & Verma, 2007). This buyer-supplier strategy is being supplanted by a more avant garde approach where the buyer-supplier relationship is characterized by long-term contracts focused on delivering performance and driving out cost for the buyer while providing satisfactory profit margins for the supplier (Randall, Pohlen, & Hanna, 2010). These performance-based service contracting strategies are referred to by a number of names such as performance-based logistics (PBL), performance-based contracting (PBC) and power-by-the-hour (PBH) with a central theme of providing an incentive structure based on multi-year contracts and shared cost avoidance (Kim, Cohen, & Netessine, 2007).

The traditional approach to post-production service contracts adopts a transactional view where a supplier's revenue and profit is generated with each service transaction. The more transactions, the more revenue and the more profit. In contrast, a performance-based strategy ties the supplier's revenue stream and profit margin to both the system performance and the cost associated with that performance. As costs go down, assuming performance within contract specification, the supplier profits increase.

One industry in which these PBC contracts are increasing is the United States Department of Defense (DoD) industry. Based upon the success of these PBC contracts the DoD has mandated performance-based contracting as the method of choice for post-production support of new systems (Vitasek & Geary, 2008). Currently the DoD is engaged in 76 performance-based contracts with another 95 scheduled in the near future (Geary & Vitasek, 2008). PBC has also been successfully employed in the commercial sector including aerospace, transportation, telecommunications, and power generation industries (Keating & Huff, 2005). By 2005, 50 countries were exploring or implementing performance-based maintenance contracts (National Cooperative Research Program, 2009). Existing practices in PBC proved its efficiency in terms of cost reductions and increases in system performance (Fowler, 2008; Kratz, 2008).

Suppliers using the traditional, transactional-based, post-production service agreements have generated satisfactory profit margins. However, this facilitates an uneasy economic imbalance between suppliers and customers. Alexander, Dayal, Dempsey, and Ark (2002) and Bundschuh and Dezvane (2003) recognize that even though after-sales support using the transactional economic model is a very profitable business for the supplier, the supplier's lack the financial incentive to invest in cost-avoidance strategies such as reliability, maintainability, and supply chain improvements. As a natural consequence of a performance-based contract, the supplier is inherently incentivized to invest in design and supply improvements to reduce out-year costs. As a result, there is often a mutually beneficial effect with the customer's maintenance reduced, the system's operational availability increased, and the supplier's profit margin increased (Kim et al., 2007).

As systems are kept in operation longer, and as support costs increase, the focus on performance-based sustainment strategies is likely to continue to gain momentum. Currently, it is commonly recognized that the operating and sustainment costs of a system often exceed 80% of the total life cycle cost of the system (Fabrycky & Blanchard, 1991). For high capital systems, these costs are substantial. For example, the expected cost to sustain the Joint Strike Fighter exceeds its development and production cost by over $250 billion (GAO, 2008). The commercial sector is equally burdened by the cost to sustain such systems. In the U.S., the airline industry spent $45 billion in 2008 on maintenance, repair, and overhaul (MRO), this is against a calculated $185 billion in revenue (ATA, 2008; Flint, 2007). These costs represent both a significant burden and a significant opportunity.

The opportunity arises from new and innovative post-production performance-based service strategies that conceptualize these sustainment cost streams as investment opportunities for the supplier and their supply chain partners. Customers must provide incentives to the suppliers for the suppliers to invest in cost-avoidance strategies. Central to any successful PBC contract is establishing a long-term relationship between a supplier and a customer (Sanders, Locke, Moore, & Autry, 2007; Sols et al., 2007). A supplier's decision to engage in a PBC with a customer, the amount of money a supplier is going to invest into cost avoidance alternatives, and the price a supplier is going to charge for its post-production services are all highly interrelated and heavily influenced by contract length.

The following fundamental research questions are addressed in our paper. Our research contact with both suppliers and buyers has showed us that these questions represent critical strategic decisions facing suppliers (and buyers) as they consider engaging in a PBC. Frequently, we have been asked to help conceptualize models that allow prediction of the economic viability of transitioning from a traditional to a performance-based service contract. That work has led us to recognize five key variables that impact the profitability, and investment decisions associated with a PBC. Those variables form the following questions.

Research Question 1: For a certain contract length, what is the optimal level of investment in cost-avoidance strategies, and what is the optimal price to charge for the post-production support service contract for an economically mutually satisfying experience for both the supplier and the customer?

Although performance-based contracting has drawn significant attention in the existing literature, most publications focus on qualitative research with a definite lack of quantitative models to assist suppliers and customers in making informed PBC decisions. Keating and Huff (2005) describe current practices in PBC and Kim et al. (2007) discuss advantages of PBC over traditional contracting. Sols et al. (2007) uncover the key characteristics of successful and unsuccessful PBC and further this research through the formulation of multi-dimensional reward and penalty schemes (Sols, Nowicki, & Verma, 2008). Nowicki, Steudel, Kumar, and Verma (2006) developed inventory allocation models in the face of PBC. However, none of the existing research has developed optimal investment and pricing strategies for performance-based contracting. This paper bridges this gap. This paper develops a decision-theoretic model that results in the optimal investment strategy, the optimal pricing strategy, and determines the optimal length of the contract and optimal reliability of the equipment, thus maximizing the supplier's profit and simultaneously satisfying the customer's performance requirements.

The paper is organized as follows. The Literature Review section reviews relevant literature on maintenance contracting, reliability, design, and pricing. The Model section develops the decision-theoretic model for performance-based contracts. The sections Model Notation Assumptions and Optimization derive the optimal investment and pricing strategies of the supplier for a given contract length. The Numerical Analysis section numerically illustrates optimal strategies and the final section concludes the paper.

Literature Review

This section presents a review of relevant literature on performance-based and traditional post-production service contracts, service pricing models, reliability, design, and the intersection of these relatable areas. While performance-based, post-production service contracting has emerged as a successful sustainment strategy in both the defense and commercial sectors (Fowler, 2008; Geary & Vitasek, 2008; Keating & Huff, 2005; Kratz, 2008), academic research in this area is only in its embryonic stage of development. Publications on performance-based contracting (PBC) mostly consist of guidebooks and good practice references found in government-issued guidebooks for suppliers (DAU, 2005a, 2005b). Existing PBC scholarship typically provides qualitative insight into current practices and implications of PBC (Kim et al., 2007; Sols et al., 2007).

The effects of PBC on the aerospace industry are discussed by Keating and Huff (2005) who suggest that PBC shifted risk from the customer to the supplier. The FCS Group for the Office of Financial Management (2005) conducted a literature review and surveyed several agencies and local jurisdictions that have implemented performance-based contracting on the best practices and trends in performance-based contracting. They identified that suppliers had a number of management issues and difficulties related to the implementation of performance-based contracting.

Few quantitative models exist in the general PBC domain and include Sols et al. (2008) who developed an n-dimensional performance model for use in a PBL arrangement. Nowicki et al.'s (2006) research examines inventory allocation under a PBL contract. Kim et al. (2007) developed a principle-agent model to study the implications of performance-based contracts by analyzing performance requirement allocation and risk sharing when a single customer is contracting with a collection of suppliers. We believe our model significantly furthers this effort by simultaneously determining the optimal investment, contract price, and contract length to maximize the supplier's profit while meeting the expectations of its customer base.

The pricing of new products and services is one of the key topics in the marketing literature (Marn, Roegner, & Zawada, 2003; Nagle & Holden, 1994; Rao, 1984). The most popular approaches to establish prices include cost-plus, return-on-investment, and perceived value pricing. The cost-plus approach sets a product's price to cover all costs associated with the product (Hanson, 2006), whereas return on investment pricing sets prices to achieve a targeted return on investment (Pride, Hughes, & Kapoor, 2008). The perceived value pricing approach is the most challenging of the three. It sets the price of a product according to a customer's perception of the product's value and requires surveying customers and inquiring about the maximal price that they are willing to pay for a product of particular quality, so called reservation price (Breidert, 2006). Optimal pricing models developed in the marketing literature are mostly focused on goods rather than on services and to the best of our knowledge there does not exist any model for optimal pricing of performance-based contracts.

Traditional maintenance contracting has been extensively studied in the literature (Levery, 2002; Sherif & Smith, 1987; Stremersch, Wuyts, & Frambach, 2001), however the existing models for traditional maintenance contracting are inapplicable for performance-based contracting since they do not simultaneously optimize pricing and investment strategies and they do not consider varying contracting periods. Murthy and Yeung (1995) used a game theoretic approach to derive optimal maintenance strategies for a customer and an independent service provider. They assumed that the customer determines the time between maintenance services and that the service provider determines the costs and the time to order spare parts. Asgharizadeh and Murthy (2000) and Murthy and Asgharizadeh (1999) use a game theoretic approach to derive their models under an assumption that a customer has to choose whether to accept a contract and to pay a fixed price or to reject the contract and to pay a cost of repair whenever equipment fails. The authors assumed that a service provider controls the price of the contract and the cost of repairs. Jackson and Pascual (2008) considered pricing of maintenance service contracts and determined the optimal number of clients to service in order to maximize the profits of a service provider.

Central to any performance-based contractual arrangement in order to properly sustain the operation of a system over time is the reliability of the system. Reliability is a dimension of quality (Murthy & Blischke, 2006) and it is defined as the probability that the product (system) will perform its intended function for a specified time period when operating under normal (or stated) environmental conditions. In the literature, the notions of reliability and quality are often used interchangeably. The majority of research on investment in product reliability optimizes the inherent trade-off between the reliability of a product and its market entry timing (Lilien & Yoon, 1990). For example, Deshmukh and Chikte (1977) presented a semi-Markov decision model for optimal funding of a product quality improvement project and time of the project termination. The authors assumed that a profit from the product is a function of the final product quality developed in comparison with that of the competing products available in the market on that date. Cohen, Eliashberg, and Ho (1996) developed a multistage model of a product quality improvement process optimizing time to market and a performance target. Levesque (2000) explored the effects of funding and its return on product quality and developed an analytical framework for optimal stopping rules for the development of the new product. Murthy, Rausand, and Virtanen (2009) developed a qualitative framework allowing manufacturers to achieve an optimal trade-off between an investment and the cost of consequences of inadequate product reliability. To the best of our knowledge, there is no research work developing a model for reliability improvement in the context of performance-based contracting.

As evident from our literature review, there is a lack of quantitative models for optimal investment and pricing strategies for suppliers offering performance-based contracts for new systems or for moving from traditional maintenance contracts to performance-based contracts for existing systems. To our knowledge, our paper is the first to develop a decision-theoretic model that optimally determines the periodic price point of a performance-based contract, the amount of money a supplier should invest in improving the reliability of the system it will contractually support, and the length of the contract between the customer and the supplier.

Model

Suppose a supplier offers a system for sale to its addressable market *M* with each potential customer having the option to engage in a post-production service contract. The salable system has an initial reliability of r_0, however, the supplier has the ability to improve the system design by investing *x* toward increasing the system's reliability according to $r(x)$, where $r(x) \geq r_0$. A customer purchasing the system is offered a post-production service contract at a fixed periodic fee *p* in exchange for a full complement of maintenance services. If the customer purchases the post-production contract, then the customer receives the system, with reliability $r(x)$, and the supplier is now responsible for the costs and risks associated with sustaining the proper operation of the system over the length (*k*) of the contract. A supplier's addressable market consists of *M* potential customers whose willingness to pay the periodic fee for the post-production service contract directly depends on the reliability of the system $r(x)$ and on the length of the service contract *k*. Let $w_{r(x),k}(v)$, $v > 0$ be the probability density function of reservation fees, that is, the maximum fee that a customer is willing to pay for the *k*-period contract if the system reliability is $r(x)$. A customer buys the post-production service contract if the supplier's actual periodic contract fee *p* is less than or equal to the customer's reservation fee. The fraction of the *M* potential customers that will engage in a post-production service contract of length *k* with the supplier is

$$W_{r,k}(p) = \int_p^\infty w_{r(x),k}(v)dv. \tag{1}$$

The total profit to the supplier, assuming the supplier invests *x* into improving the reliability of its system's design, is

$$\Pi(x,p,k) = M\sum_{j=1}^{l}\frac{1}{(1+i)^j}(p - f(r(x)))\int_p^\infty w_{r(x),k}(v)dv - x \quad , \tag{2}$$

where *p* is a periodic contract fee, *i* is an interest rate, and $f((r(x))$ is the total cost of all system failures for a single period within a *k*-period contract given that the system has a reliability of $r(x)$.

Model Notation and Assumptions

Table 1

M	number of potential customers.
k	length of a contact.
m	number of missions in a single time period of a contract of length k.
r_0	initial reliability of the system for the mission time t_m.
$r(x)$	reliability of the system for a cost avoidance investment of x.
γ	marginal investment parameter.
$f(r(x))$	total cost of all system failures for a single period, given that the system has a reliability $r(x)$.
μ_c	average cost per failure.
σ_c	standard deviation of the cost per failure.
p	periodic contract fee.
i	interest rate.
d	discount per period expected by customers.
λ	maximal fee that customers are willing to pay for the single-period contract if $r(x)$ = 1.
$w_{r(x),k}$	probability density function of customers reservation fees.
$W_{r(x),k}(p)$	fraction of customers that will engage in the k-period contract with the periodic fee equal to p and the reliability of the system is $r(x)$.
$\Pi(x,p,k)$	total profit to the supplier when investing capital x into the system reliability design for a k-period post-production contract with periodic fee p.

The new, decision-theoretic post-production service model developed herein is greatly influenced by the reliability of the system the supplier is contracted to sustain, the cost to the supplier each time a maintenance action is required, the supplier's total ownership cost of a system failure, and the willingness of a customer to engage in a post-production service contract with the supplier. Each of these variables are discussed below, highlighting the defining assumptions and key interrelationships:

Notation

Let us make the following four assumptions, denoted by (A1)-(A4):

(A1) The system reliability r depends on cost avoidance investment x in the following way:

$$r(x) = r_o + (1 - r_o)\left(1 - \frac{1}{x/\gamma + 1}\right) = \frac{x + r_o\gamma}{x + \gamma}, \tag{3}$$

where $\gamma > 0$ is a marginal investment parameter, defined as the marginal investment required to achieve an incremental improvement of system reliability. The function $r(x)$ satisfies the assumption regarding the initial reliability of the equipment ($r(0) = r_0$). The signoid shape of the curve $r(x)$ describes the relationship between system reliability and investment observed in reality fairly well (Levesque, 2000).

(A2) The cost per failure is a normally distributed random variable with the mean μ_c and variance σ_c^2.

(A3) The expected cost of all system failures per period decreases with reliability improvements is $f(r(x)) = cm(1-r(x))$, where m is the number of missions in a single time period.

(A4) The customers' reservation fees follow the triangular distribution:

$$w_{r(x),k}(v) = \begin{cases} \dfrac{(\lambda(1-d(k-1))r-p)^2}{(\lambda(1-d(k-1))r)^2}, & 0 \le p \le \lambda(1-d(k-1))r \\ 0, & o.w. \end{cases} , \tag{4}$$

where λ is a maximal fee that customers are willing to pay for the contract if reliability of the equipment will be improved to $r(x) = 1$ and d is a discount per period expected by customers if they buy a multi-period contract. The use of a triangular distribution to represent reservation fees is consistent with the current state of the pricing literature (Kirman, Schulz, Hardle, & Werwatz, 2005).

Optimization

The goal of the supplier is to identify an optimal investment x^*, optimal periodic contract fee p^* and optimal contract length k^* that maximize the supplier's expected profit $E[\Pi(x,p,k)]$ from a k-period contract ($k = 1,...,n$):

$$E\left[\Pi(x^*, p^*, k^*)\right] = \max_{k=1,\dots,n} E\left[\Pi(x^*, p^*, k)\right], \tag{5}$$

where,

$$E\left[\Pi(x^*, p^*, k^*)\right] = \max_{\{x,p\}\in F_{x,p}} E\left[\Pi(x, p, k)\right], \tag{6}$$

with a set of feasible solutions:

$$F_{x,p} = \{\{x,p\} \mid x > 0, 0 \le p \le \lambda)1-d(k-1))r\}. \tag{7}$$

where the upper bound for the price follows from triangularly distributed customers' reservation prices. Under the assumptions (A1)-(A4), an expected profit is given by

$$E\left[\Pi(x,p,k)\right] = \begin{cases} \dfrac{MI_k(p(x+\gamma)-\mu_c m(1-r_o)\gamma)(p(x+\gamma)-\lambda D_k(x+r_o\gamma))^2}{\lambda^2 D_k^2 (x+r_o\gamma)^2 (x+\gamma)} - x, & 0 \le p \le \lambda D_k r(x) \\ 0, & o.w. \end{cases} \tag{8}$$

where, $D_k = (1 - d(k - 1))$ and $I_k = (1 + i - (1 + i)^{-k})/i$.

The optimal investment x^* and the optimal periodic fee p^* for the k-period contract are either critical points determined from the first order necessary conditions:

$$\left.\frac{\partial E\left[\Pi(x,p,k)\right]}{\partial x}\right|_{(x^*,p^*,k)} = 0$$

and $$\left.\frac{\partial E\left[\Pi(x,p,k)\right]}{\partial p}\right|_{(x^*,p^*,k)}=0, \tag{9}$$

or belong to the boundary of the feasible set F_{xp}. With Equation 8, Equation 9 reduces to

$$p=\frac{2\mu_c m(1-r_o)\gamma+\lambda D_k X}{3(X-\gamma(1-r_o))}$$

and

$$4M\gamma I_k(1-r_o)\left(X\lambda D_k - cm(1-r_o)\gamma\right)^2(cm(3X+2(1-r_o)\gamma)+X\lambda D_k)-27X^3\lambda^2 D_k^{\,2}(X+(1+r_o)\gamma)^2=0$$

where, $X = x + r_0\gamma$. If (x^*,p^*) is a critical point it satisfies the second order sufficient conditions:

$$\left.\frac{\partial^2 E\left[\Pi(x,p)\right]}{\partial^2 x}\right|_{(x^*,p^*)}<0, \quad \text{and} \quad \left.\frac{\partial^2 E\left[\Pi(x,p)\right]}{\partial^2 p}\right|_{(x^*,p^*)}<0, \tag{10}$$

and

$$\left.\frac{\partial^2\Pi(x,p)}{\partial^2 x}\frac{\partial^2\Pi(x,p)}{\partial^2 p}-\frac{\partial^2\Pi(x,p)}{\partial x\partial p}\frac{\partial^2\Pi(x,p)}{\partial p\partial x}\right|_{(x^*,p^*)}>0, \tag{11}$$

The optimal solution (x^*, p^*) is obtained numerically for all $k = 1,\dots,n$ and the optimal contracting period k^* follows from Equation 5.

Numerical Analysis

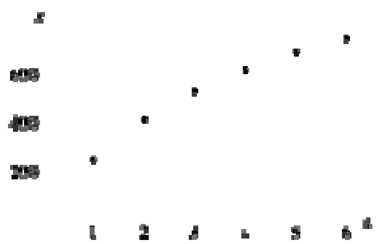
This section analyzes how the optimal investment x^*, optimal contract fee p^*, optimal contract length k^*, reliability $r(x^*)$, and the expected profit $\Pi^* = E[\Pi(x^*,p^*,k)]$ depend on parameters d, λ, μ_c, r_0, M and γ.

Suppose a supplier of airplane engines plans to introduce a performance-based post-production service option to a market consisting of 60 potential customers (M = 60). The maximal periodic fee that customers are willing to pay for the post-production maintenance service contract is \$100,000 (λ = 100). Customers expect a 7% discount per period if they subscribe to a multi-period contract (d = 0.07). The initial reliability of the engines is 0.7 (r_0 = 0.7) and at least a \$100, 000 investment is required to improve the reliability of the engines up to r_0 + 1/2(1 -r_0) = 0.85 (γ = 100). Let the periodic interest rate be equal to 5% (i = 0.05). The expected cost per failure is \$20,000 (μ_c=20) and the variance of the cost per failure is \$4,000 (σ_c=4). Assume that a period consists of 10 missions (m = 10). Table 2 summarizes the parameters considered in this example.

Table 2. Baseline Example

Parameter	d	M	**λ**	**r_0**	**γ**	**μ_c**	**σ_c**	**m**	**i**
Value	0.07	60	100	0.7	100	20	10	10	0.05

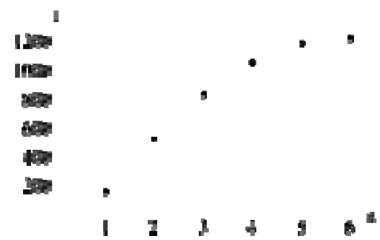
The optimal investment and the optimal contract fee for each *k*-period contract (*k* = 1,...,6) are obtained from Figures 1(a) and 1(c). The results, as illustrated through this example, suggest that the longer the contract length, the higher the optimal investment and the lower the optimal periodic contract fee. Figures 1(b) and 1(d) show that a longer contract length results in a system that is delivered with a higher reliability to the customer and provides an even greater profit to the supplier. Herein lies the economic win-win for both the supplier and customer and provides the necessary mechanisms to properly incentivize the supplier to invest in cost avoidance strategies. In this example, a 6-period contract is best with an optimal investment of $751,302 and the optimal periodic contract fee is $25,602. This contract results in reliability equal to 0.965 and the expected total profit of the supplier is $1,227, 210.



(a) Optimal investment (x) vs contract length (k)

(b) Reliability r(x) vs contract length (k)

(c) Optimal contract fee (p) vs contract length (k)



(d) Profit (π) vs contract length (k)

Figure 1. Optimal Investment, Reliability, Periodic Contract Fee and Profit as Functions of Contract Length

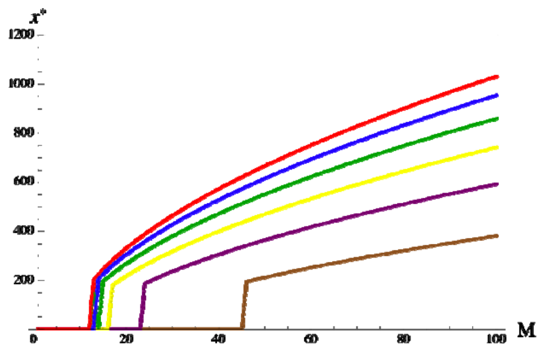
Parameters in Table 2 may vary due to different economic conditions. The remainder of this section discusses the sensitivity of the optimal (for the considered example) results on the parameters of the model. Understanding the sensitivity of these parameters is central to the contractual negotiation process for both the supplier and the customer. Figures 2–7 show x^*, p^*, Π^* and k^* as functions of the discount per period, *d*; market size, *M*; customers' willingness to pay, λ; initial reliability, r_0; marginal investment parameter, γ; and the expected cost per failure, μ_c. Table 3 summarizes how the optimal contract's length depends on variations of the parameters $d, M, \lambda, r_0, \gamma, \mu_c$.

Table 3. Sensitivity of the Optimal Contract's Length on the Model Parameters

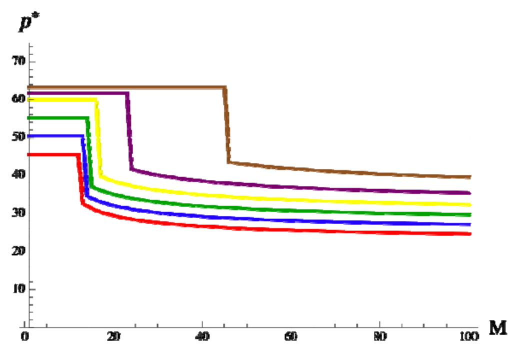
Parameter	*k**=6	*k**=5	*k**=4	*k**=3	*k**=2	*k**=1	*No contract*
d	[0, 0.08]	(0.08, 0.1]	(0.1, 0.13]	(0.13, 0.18]	(0.18, 0.33]	[0.33, 0.4]	
M	[38, 100]	[20, 38)					[0,20)

λ	[80, 150]	[60, 80]						
r_0	[0.5, 0.9]							
γ	[0,170)	[170,380)						[380,+∞)
μ_c	[0, 34)	[34,40)						

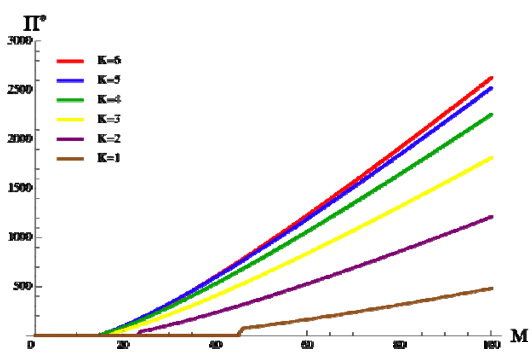
Optimal investment and contract length are increasing functions of the market size, whereas the optimal periodic contract fee is a decreasing function of the market size (see Figure 2). Moreover, a certain critical market size, which depends on the contract's length, is required for profitability of a contract. For example, it is unprofitable to provide 1-period contracts if the potential market has less than 46 customers and it is unprofitable to provide 6-period contracts if there are less than 14 potential customers on the market, see Figure 3(c). This has the following interpretation. Customers are willing to pay higher fees as the reliability of engines improves. Consequently, the supplier has to invest as much as possible in reliability improvement. However, if the supplier invests large capital in reliability and the market size is small, the supplier may not break even. Thus the optimal investment increases gradually with market size.



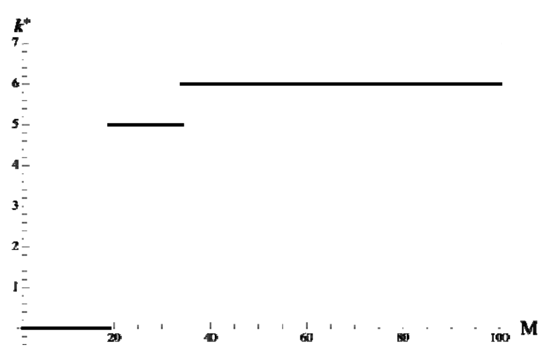
(a) Optimal investment



(b) Optimal periodic contract fee



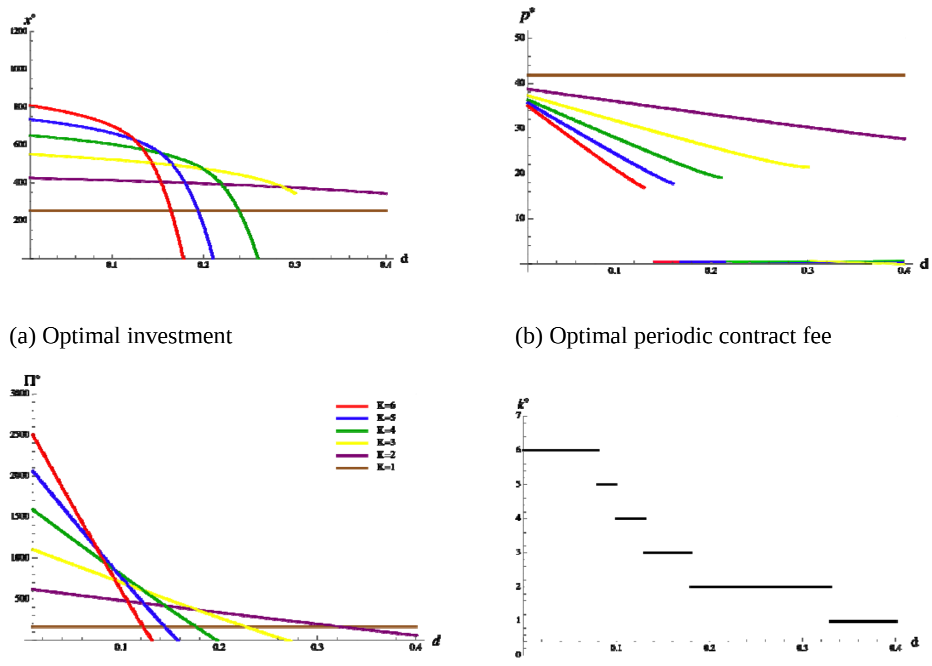
(c) Profit



(d) Optimal contract's length

Figure 2. Optimal Investment, Periodic Contract Fee, Profit and Optimal Contract's Length as Functions of the Market Size

The optimal investment, the contract's periodic fee, and length are decreasing functions of the discount per period expected by customers (see Figure 3). Although, in general, longer contracts are more profitable, the supplier should offer shorter contracts if the discount per period is high.



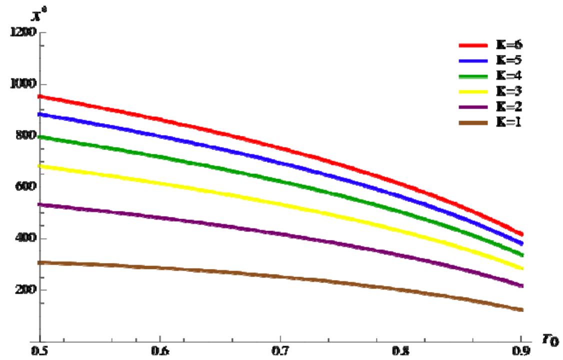
(a) Optimal investment (b) Optimal periodic contract fee

(c) Profit (d) Optimal contract's length

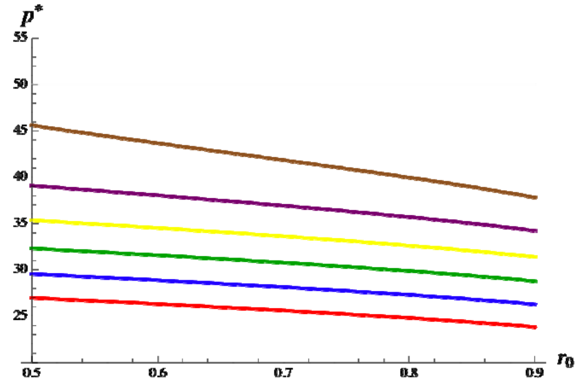
Figure 3. Optimal Investment, Periodic Contract Fee, Profit and Optimal Contract's Length as Functions of the Discount Expected by Customers

The optimal investment, periodic contract fee, and the contract's length are increasing functions of the maximal price that customers are willing to pay for a single-period contract (see Figure 4). In other words, the more customers are willing to pay, the higher fees the supplier should charge.

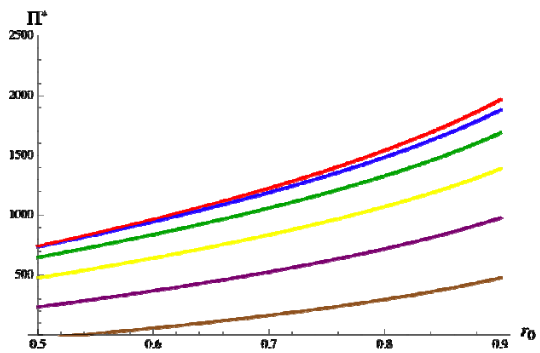
The optimal investment and periodic contract fee are decreasing functions of the initial reliability (see Figure 5). The higher the initial reliability the less the supplier has to invest to achieve a targeted level of reliability and consequently the lower the optimal contract fee.



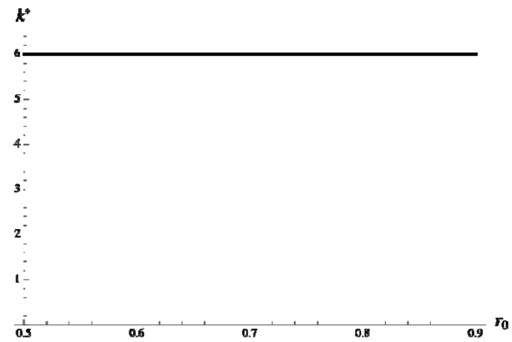
(a) Optimal investment



(b) Optimal periodic contract fee



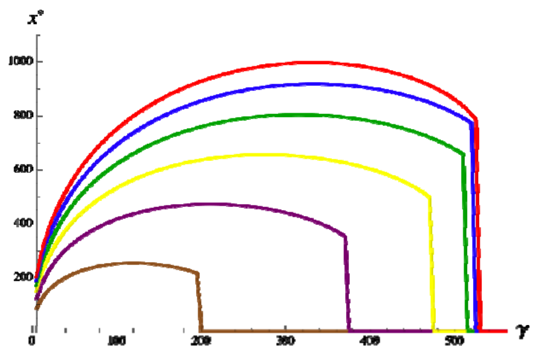
(c) Profit



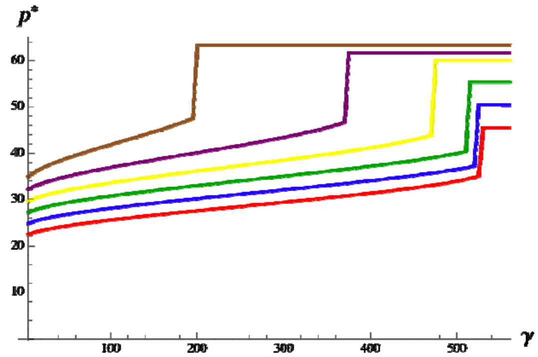
(d) Optimal contract's length

Figure 5. Optimal Investment, Periodic Contract Fee, Profit and Optimal Contract's Length as Functions of the Initial Reliability

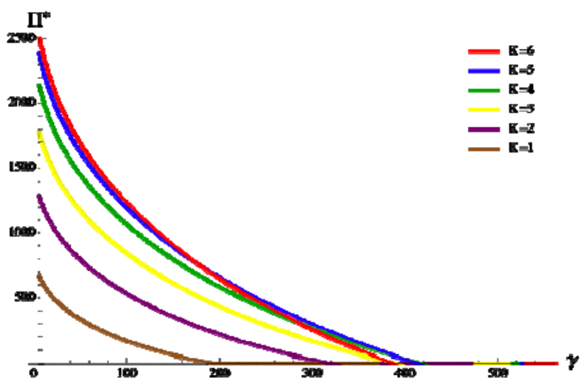
The optimal investment and periodic contract fee increase as the marginal investment parameter increases (see Figure 6). However, for each contract length there exists a marginal investment threshold where it is unprofitable for the supplier to invest in reliability improvement. For example, if γ >200, it is unprofitable to invest in reliability improvements for 1-period contracts and if γ >380, it is unprofitable to invest in reliability improvements for 2-period contracts. Thus, the threshold level rises with the contract's length.



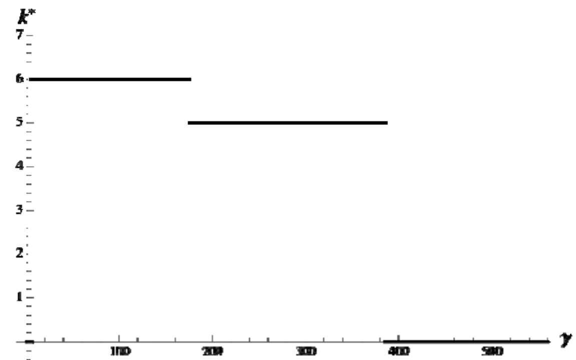
(b) Optimal investment



(c) Optimal periodic contract fee



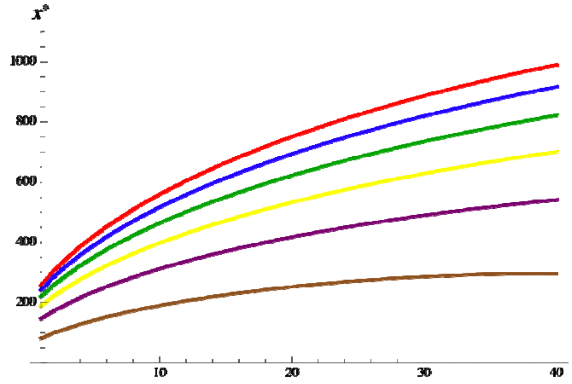
(d) Profit



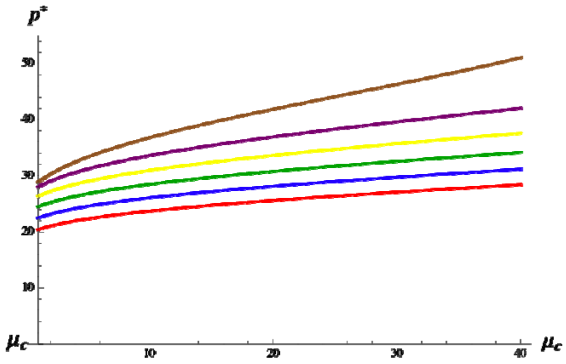
(e) Optimal contract's length

Figure 6. Optimal Investment, Periodic Contract Fee, Profit and Optimal Contract's Length as Functions of the Marginal Investment Parameter

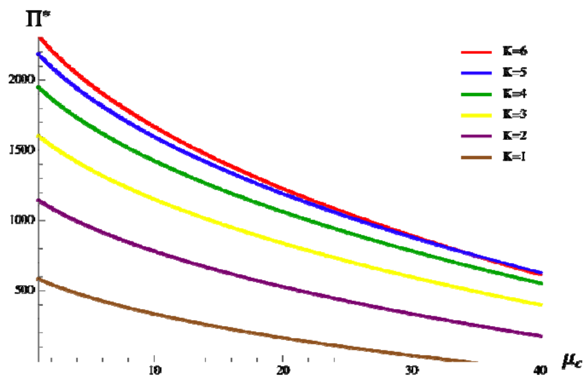
The optimal investment and periodic contract fee are increasing functions of the expected cost per failure (see Figure 7). If the cost per failure is high, the supplier needs to invest in reliability as much as possible in order to reduce the number of future failures and consequently avoid future costs. More failures are likely to occur during longer contracts. Thus, the optimal contract's length is a decreasing function of the expected cost per failure.



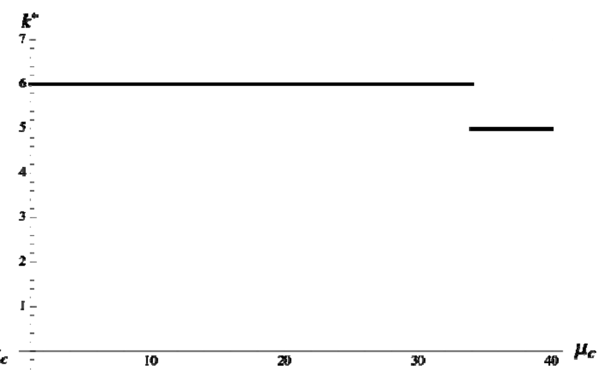
(b) Optimal investment



(c) Optimal periodic contract fee



(d) Profit



(e) Optimal contract's length

Figure 7. Optimal Investment, Periodic Contract Fee, Profit and Optimal Contract's Length as Functions of the Expected Cost Per Failure

In summary, the following conclusion can be drawn:

- The optimal investment is an increasing function of the expected cost per failure, the market size, and the customers' willingness to pay, but it is a decreasing function of the initial reliability.
- The optimal periodic contract fee is an increasing function of the contract's length, the customers' willingness to pay, and an expected cost per failure, but it is a decreasing function of the initial reliability and market size.
- Longer post-production service contracts require higher optimal investments but provide higher system reliability.
- Optimal contract length is a decreasing function of the discount per period, the expected cost per failure, and the marginal investment parameter, and it is an increasing function of the market size and the maximal price that customers are willing to pay for a single-period contract.

Conclusions

As performance-based contracts (PBC) continue to gain momentum, it is important for suppliers to determine the right price to charge in order to capture business to provide service to its own systems or other systems it is capable of sustaining. This paper develops a decision-theoretic model to assist suppliers in defining their investment and pricing strategies for performance-based, post-production service contracts. To our knowledge we are the first to develop, under a PBC, a mathematical model and corresponding solution to determine the optimal investment, contract price, and contract length that maximizes the supplier's profit while meeting the expectations of its customer base.

Of particular interest is gaining insight into the underlying motivation for a supplier to engage in a PBC with a customer, or collection of customers, or a customer's willingness to enter into a PBC with a supplier. Our findings suggest that these decisions are heavily influenced by the contract length, the supplier's level of cost avoidance investment, and the periodic contract fee the supplier offers to its addressable market.

Numerical examples analyze the optimal contract length, investment, system reliability, and optimal periodic fee with respect to the initial system reliability, customers' willingness to pay, the expected cost per failure, and other parameters of the model. The findings from this numerical example suggest that there is a formidable tradeoff space in determining, first and foremost, if a supplier should offer a PBC to its customer base, and if a PBC is offered what price should be offered. The price offering is heavily influenced by the reliability of the system the supplier is offering to service, the length of the contract, and the amount of money the supplier will invest into cost avoidance strategies such as reliability and supply chain improvements.

We believe this is just the beginning of an area of research that focuses on managerial decisions at the intersection of system design, supply chains, and sustainment. Cost avoidance strategies run the gambit of improving the reliability of a system to capital investment into spares to satisfy a customer's requirements. Among other research questions is how to optimally allocate funds among competing cost avoidance alternatives? As it relates to PBC, a future area of research is to determine how to invest in these competing and sometimes complimentary cost avoidance alternatives in order to increase the likelihood of contract capture and to further increase profit.

References

Air Transport Association (ATA). (2008). Air Transport Association 2008 economic report: Connecting/protecting our planet. In J. May (Ed.), *Air Transport Association economic report* (p. 32). Washington, DC: Author.

Alexander, W. S., Dayal, S., Dempsey, J., & Ark, J. V. (2002). The secret life of factory service centers? *The McKinsey Quarterly, 3,* 107–116.

Asgharizadeh, E., & Murthy, D. N. P. (2000). Service contracts: A stochastic model. *Mathematical and Computer Modelling, 31*(10–12).

Breidert, C. (2006). Estimation of willingness-to-pay theory, measurement, and application. *Willingness-to-Pay (WTP) in Marketing*.

Bundschuh, R., & Dezvane, T. (2003). How to make after sale services pay off. *The McKinsey Quarterly, 4,* 116–127.

Cohen, M. A., Eliashberg, J., & Ho, T. H. (1996). New product development: The performance and time-to-market tradeoff. *Management Science, 42*(2), 173.

Defense Acquisition University (DAU). (2005a). *Guidebook*. Retrieved from http://akss.dau.mil/dag

Defense Acquisition University (DAU). (2005b). *Performance based logistics: A program manager's product support guide*. Defense Acqusition University Press.

Deshmukh, S. D., & Chikte, S. D. (1977). Dynamic investment strategies for a risky R and D project. *Journal of Applied Probability, 14*(1), 144–152.

Fabrycky, W. J., & Blanchard, B. S. (1991). *Life cycle cost and economic analysis*. New Jersey: Prentice Hall.

FCS Group for Office of Financial Management. (2005). *Best practices and trends in performance based contracting*.

Flint, P. (2007). Balancing Act. *Air Transport World, 44*(11), 47–54.

Fowler, R. (2008). Performance based life cycle product support—The new PBL. *North American Defense Logistis Conference*.

Geary, S., & Vitasek, K. (2008). *Performance-based logistics: A contractor's guide to life cycle product support management*. Bellevue, WA: Supply Chain Visions.

GAO. (2008). *Joint strike fighter recent decisions by Department of Defense add to program risks*. Washington DC: Author.

Hanson, W. (2006). The dynamics of cost-plus pricing. *Managerial Decision Economics, 13*(2), 149–161.

Jackson, C., & Pascual, R. (2008). Optimal maintenance service contract negotiation with aging equipment. *European Journal of Operational Research, 189*.

Keating, S., & Huff, K. (2005). Managing risk in the new supply chain [Article]. *Engineering Management, 15*(1), 24–27.

Kim, S.-H., Cohen, M. A., & Netessine, S. (2007). Performance contracting in after-sales service supply chains. *Management Science, 53*(12), 1843–1858.

Kirman, A., Schulz, R., Hardle, W., & Werwatz, A. (2005). Transactions that did not happen and their influence on prices. *Journal of Economic Behavior and Organization, 56*(4), 567–591.

Kratz, L. (2008). *Performance based life cycle product support—The new PBL*. Paper presented at the North American Defense Logistics Conference, Crystal City, VA.

Levery, M. (2002). Making maintenance contracts perform. *Engineering Management Journal, 12*(2), 76–82.

Levesque, M. (2000). Effects of funding and its return on product quality in new ventures. *IEEE Transactions on Engineering Management, 47*(1), 98–105.

Lilien, G. L., & Yoon, E. (1990). The timing of competitive market entry: An exploratory study of new industrial products. *Management Science, 36*(5), 568.

Marn, M. V., Roegner, E. V., & Zawada, C. C. (2003). Pricing new products. *The McKinsey Quarterly, 3*, 40–49.

Murthy, D. N. P., & Asgharizadeh, E. (1999). A stochastic model for service contracts. *International Journal of Reliability Quality and Safety, 5*, 29–45.

Murthy, D. N. P., & Blischke, W. R. (2006). *The warranty management and product manufacture*. London, UK: Springer.

Murthy, D. N. P., Rausand, M., & Virtanen, S. (2009). Investment in new product reliability. *Reliability Engineering and System Safety, 94*.

Murthy, D. N. P., & Yeung, V. (1995). Modelling and analysis of maintenance service contracts. *Mathematical and Computer Modeling, 22*(10–12), 219–225.

Nagle, T. T., & Holden, R. K. (1994). *The strategy and tactics of pricing: A guide to profitable decision making*. Englewood Cliffs, NJ: Prentice Hall.

National Cooperative Research Program. (2009). Performance-based contracting for maintenance.

Nowicki, D., Steudel, H., Kumar, U., & Verma, D. (2006). Spares provisioning under performance-based logistics contract: Profit-centric approach. *Journal of Operational Research Society*.

Pride, W., Hughes, R., & Kapoor, J. (2008). *Business: South-Western cengago learning*.

Randall, W. S., Pohlen, T. L., & Hanna, J. B. (2010). Evolving a theory of performance based logistics using insights from service dominant logic. *Journal of Business Logistics,* Forthcoming.

Rao, V. R. (1984). Pricing research in marketing: The state of the art. *Journal of Business, 57*(1).

Sanders, N. R., Locke, A., Moore, C. B., & Autry, C. W. (2007). A multidimensional framework for understanding outsourcing arrangements [Article]. *Journal of Supply Chain Management: A Global Review of Purchasing & Supply, 43*(4), 3–15. doi: 10.1111/j.1745-493X.2007.00037.x

Sherif, Y. S., & Smith, M. L. (1987). Optimal maintenance models for systems subject to failure—A review. *Naval Research Logistics Quarterly, 34*, 47–74.

Sols, A., Nowicki, D., & Verma, D. (2007). Defining the fundamental framework of an effective performance-based logistics contract. *Engineering Management Journal, 19*(2), 40–50.

Sols, A., Nowicki, D., & Verma, D. (2008). n-Dimensional effectiveness metric-compensating reward scheme in performance-based logistics contracts. *Systems Engineering, 11*(2), 93–106.

Stremersch, S., Wuyts, S., & Frambach, R. T. (2001). The purchasing of full-service contracts: An exploratory study within the industrial maintenance market. *Industrial Marketing Management, 30*(1), 1–12.

Vitasek, K., & Geary, S. (2008, June). Performance-based logistics redefines Department of Defense procurement. *World Trade Magazine, June,* 62–65.

Acknowledgements

This material is based upon work supported by the Naval Postgraduate School Acquisition Research Program under Grant No. N00244-10-1-0074.