NPS-AM-11-C8P24R03-079

EXCERPT FROM THE PROCEEDINGS

OF THE

EIGHTH ANNUAL ACQUISITION RESEARCH SYMPOSIUM THURSDAY SESSIONS VOLUME II

On a Quantitative Definition of Affordability

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Published: 30 April 2011

Prepared for the Naval Postgraduate School, Monterey, California 93943

ACQUISITION RESEARCH PROGRAM
GRADUATE SCHOOL OF BUSINESS & PUBLIC POLICY
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The research presented at the symposium was supported by the Acquisition Chair of the Graduate School of Business & Public Policy at the Naval Postgraduate School.

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Preface & Acknowledgements

During his internship with the Graduate School of Business & Public Policy in June 2010, U.S. Air Force Academy Cadet Chase Lane surveyed the activities of the Naval Postgraduate School's Acquisition Research Program in its first seven years. The sheer volume of research products—almost 600 published papers (e.g., technical reports, journal articles, theses)—indicates the extent to which the depth and breadth of acquisition research has increased during these years. Over 300 authors contributed to these works, which means that the pool of those who have had significant intellectual engagement with acquisition issues has increased substantially. The broad range of research topics includes acquisition reform, defense industry, fielding, contracting, interoperability, organizational behavior, risk management, cost estimating, and many others. Approaches range from conceptual and exploratory studies to develop propositions about various aspects of acquisition, to applied and statistical analyses to test specific hypotheses. Methodologies include case studies, modeling, surveys, and experiments. On the whole, such findings make us both grateful for the ARP's progress to date, and hopeful that this progress in research will lead to substantive improvements in the DoD's acquisition outcomes.

As pragmatists, we of course recognize that such change can only occur to the extent that the potential knowledge wrapped up in these products is put to use and tested to determine its value. We take seriously the pernicious effects of the so-called "theory–practice" gap, which would separate the acquisition scholar from the acquisition practitioner, and relegate the scholar's work to mere academic "shelfware." Some design features of our program that we believe help avoid these effects include the following: connecting researchers with practitioners on specific projects; requiring researchers to brief sponsors on project findings as a condition of funding award; "pushing" potentially high-impact research reports (e.g., via overnight shipping) to selected practitioners and policy-makers; and most notably, sponsoring this symposium, which we craft intentionally as an opportunity for fruitful, lasting connections between scholars and practitioners.

A former Defense Acquisition Executive, responding to a comment that academic research was not generally useful in acquisition practice, opined, "That's not their [the academics'] problem—it's ours [the practitioners']. They can only perform research; it's up to us to use it." While we certainly agree with this sentiment, we also recognize that any research, however theoretical, must point to some termination in action; academics have a responsibility to make their work intelligible to practitioners. Thus we continue to seek projects that both comport with solid standards of scholarship, and address relevant acquisition issues. These years of experience have shown us the difficulty in attempting to balance these two objectives, but we are convinced that the attempt is absolutely essential if any real improvement is to be realized.

We gratefully acknowledge the ongoing support and leadership of our sponsors, whose foresight and vision have assured the continuing success of the Acquisition Research Program:

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- Program Executive Officer Integrated Warfare Systems
- Office of the Assistant Secretary of the Air Force (Acquisition)
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We also thank the Naval Postgraduate School Foundation and acknowledge its generous contributions in support of this Symposium.

James B. Greene, Jr.
Rear Admiral, U.S. Navy (Ret.)

Keith F. Snider, PhD
Associate Professor

Panel 24 – The Other "Big A": Coming to Grips with Affordability

Thursday, May 12, 2011	
3:30 p.m. – 5:00 p.m.	**Chair: Reuben Pitts III,** President, Lyceum Consulting, LLC **Discussant: Brigadier General Michael E. Williamson, US Army,** Joint Program Executive Officer for the Joint Tactical Radio System ***Military Cost-Benefit Analysis: Introducing Affordability in Vendor Selection Decisions*** Francois Melese, Anke Richter, and Jay Simon, NPS ***On a Quantitative Definition of Affordability*** Charles LaCivita and Kent Wall, NPS

Reuben S. Pitts III—President, Lyceum Consulting. Mr. Pitts joined the Naval Weapons Lab in Dahlgren, VA, in June 1968 after graduating from Mississippi State University with a BSME. His early career was spent in ordnance design and weapons systems. He subsequently served on the planning team to reintroduce the Navy to Wallops Island, VA, currently a multiple ship combat, over-the-water weapons testing lab for Surface Ship Combat Systems, Fighter Aircraft, and live missile firings. His outstanding service as the deployed Science Advisor to Commander, U.S. Sixth Fleet was recognized with the Navy's Superior Civilian Service (NSCS) Award and the Navy Science Assistance Program Science Advisor of the Year Award.

Mr. Pitts was selected to lead the technical analysis team in support of the formal JAG investigation of the downing of Iran Air Flight 655 by USS *Vincennes*, and participated in subsequent briefings to CENTCOM, the Chairman of the Joint Chiefs, and the Secretary of Defense. As Head, Surface Ship Program Office and Aegis Program Manager, Mr. Pitts was awarded a second NSCS, the James Colvard Award, and the John Adolphus Dahlgren Award (Dahlgren's highest honor) for his achievements in the fields of science, engineering, and management. Anticipating the future course of combatant surface ships, Mr. Pitts co-founded the NSWCDD Advanced Computing Technology effort, which eventually became the Aegis/DARPA-sponsored High Performance Distributed Computing Program; the world's most advanced distributed real-time computing technology effort. That effort was the foundation for the Navy's current Open Architecture Initiative.

In 2003 Mr. Pitts accepted responsibility as Technical Director for PEO Integrated Warfare Systems (IWS), the overall technical authority for the PEO. In September of that year, he was reassigned as the Major Program Manager for Integrated Combat Systems in the PEO. In this position, he was the Program Manager for the Combat Systems and Training Systems for all U.S. Navy Surface Combatants, including Aircraft Carriers, Cruisers, Destroyers, Frigates, Amphibious Ships, and auxiliaries. In July, 2006, Mr. Pitts returned to NSWCDD to form and head the Warfare Systems Department. While in this position, he maintained his personal technical involvement as the certification official for Surface Navy Combat Systems. He also served as Chair of the Combat System Configuration Control Board and Chair of the Mission Readiness Review for Operation Burnt Frost, the killing of inoperative satellite USA 193.

Mr. Pitts has been a guest speaker/lecturer/symposium panelist at many NAVSEA-level and DoD symposiums, conferences and at the Naval Postgraduate School, the Defense Systems Management College, and the National Defense University. For 19 years Mr. Pitts was the sole certification authority of all Aegis Combat System computer programs for fleet use. He retired from the U.S. Civil Service in September 2008, with over 40 years of service to the Navy.

Brigadier General Michael E. Williamson, US Army—Joint Program Executive Officer for the Joint Tactical Radio System.

General Williamson was born in Tucson, Arizona. He was commissioned at the University of Maine as a Second Lieutenant in the Air Defense Artillery in 1983. His assignments include service as the Automation Officer for the 32nd AADCOM in Darmstadt Germany. He then served as a Chaparral Platoon Leader, Vulcan Platoon Leader, Maintenance Officer and Executive Officer in C Battery, 108th Brigade, Hahn Air Force Base, Germany. After attending the Air Defense Artillery Advance Course, he served as the Chief, Forward Area Air Defense Weapons, Development Branch at Fort Bliss, Texas. He then commanded B Battery, 3/1 ADA (Hawk) in the 11th Brigade at Fort Bliss and also in the 31st ADA Brigade at Fort Hood, Texas. After completing command, he served as the Assistant S-3 in the 31st ADA Brigade.

His acquisition experience began as Sr. Military Software Analyst at NATO's military headquarters in Mons, Belgium. He then served as the Associate Director, Battle Command Battle Lab at Fort Leavenworth, Kansas. After attending Command and General Staff College, he served as the Chief of Information Technology, Acquisition Career Management, within the Office of the Assistant Secretary of the Army for Acquisition Logistics and Technology. He was then selected as a Congressional Fellow and served as a legislative assistant to a Member of Congress. After completing the fellowship, General Williamson served as the Product Manager for the Global Command and Control System-Army, and then as the Acquisition Military Assistant to the Secretary of the Army. He served as Commander of Software Engineering Center-Belvoir (SEC-B), He was then assigned as the Project Manager, Future Combat System (Brigade Combat Team) Network Systems' Integration within Program Manager, Future Combat System (Brigade Combat Team). He then served as the Director of Systems Integration, within the Office of the Assistant Secretary of the Army for Acquisition Logistics and Technology. Prior to his current assignment, General Williamson served as the Deputy Program Manager, Program Executive Office, Integration.

General Williamson's awards and decorations include the Legion of Merit with two Oak Leaf Clusters; the Meritorious Service Medal with 2 Oak Leaf Clusters; the Joint Service Commendation medal, the Army Commendation Medal with two Oak Leaf Clusters, the Joint Service Achievement Medal, the Army Achievement Medal with two Oak Leaf Clusters, the Army Superior Unit Award, the National Defense Service Medal with Bronze Star, the Global War on Terrorism Service Ribbon, the Army Service Ribbon, the Overseas Ribbon and the Army Staff Identification Badge.

General Williamson's education includes a Bachelor of Science from Husson College in Business Administration, a Masters of Science in Systems Management from the Naval Postgraduate School and a PhD in Business Administration from Madison University. He also has graduate certificates in Public Policy from the JFK School of Government, Harvard University and the Government Affairs Institute at Georgetown University. He is a graduate of the Army Command and General Staff College, a graduate of the Advanced Management Program at the Harvard Business School and was a Senior Service College Fellow at the University of Texas at Austin. He is Level III certified in Program Management and Communications and Computers.

On a Quantitative Definition of Affordability

Charles LaCivita—Executive Director, USPTC Program Office, NPS. Dr. LaCivita joined the faculty of the Naval Postgraduate School in 1985. Previously, he was Assistant Professor of Economics at the University of North Carolina at Greensboro. At NPS, he has served as the Assistant Director for Academic Programs, the Executive Director of the Defense Resources Management Institute (DRMI), the Chair of the Global Public Policy Academy Group (GPPAG), and his most recent appointment as USPTC Program Office Executive Director. His current research concerns the relationship between accounting costs and economic costs and their use in promoting more efficient management of defense resources. Professor LaCivita earned his doctorate in Economics from the University of California at Santa Barbara. [clacivita@nps.edu]

Kent Wall—Professor, NPS. Dr. Wall attended the University of Minnesota, earning a PhD in Control Sciences. After completing his studies he was awarded two postdoctoral fellowships in England, the first with the University of Manchester and the second with the University of London. While in the UK, he did lecturing at H.M. Treasury, the Bank of England, Queen Mary College, Imperial College, London School of Economics, and London Business School. He returned to the U.S. as a Research Associate with the National Bureau of Economic Research in Cambridge, MA. Before coming to the Naval Postgraduate School he was an Associate Professor of Systems Engineering at the University of Virginia. His research interests focus on the development of quantitative aids in decision making. He has published his work in many scholarly journals, including the *IEEE Trans*. *Automatic Control*, *Automatica*, *Proceedings of the IEE*, *Communications in Statistics*, *Jour. Business and Economic Stats.*, *Jour. Time Series Anal.*, *Jour. of Econometrics* and *JASA*. He joined the faculty in August 1985 and served as Assistant Director for Academic Programs from 1993–1998. In 1995 he was invited to present a special course in time series modeling at the University of Paris IX (Dauphine). [kdwall@nps.edu]

Abstract

This paper introduces a definition of affordability based on the microeconomic theory of the consumer. We replace utility maximization with effectiveness maximization and discuss our conceptualization in terms of a cost-effectiveness framework. We convert our original ideas into a more useful degree (amount) of affordability (i.e., we ask not “Is it affordable?” but “How affordable is it?”). This allows us to attach meaning to, and interest in, the concept of an affordability index—or the measurement of the degree of affordability.

Introduction

There is currently intense debate over whether various government programs (e.g., health care, defense, and the environment) are affordable. There are also questions about the long-term affordability of Social Security and Medicare. Given that affordability is at the forefront of many of these programs, it is imperative that we can define and quantify it. Affordability is a concept that everyone seems to understand, but that everyone also has trouble precisely defining and even more trouble quantifying. Webster’s defines affordability as “the ability to manage or to bear the cost of without serious loss or detriment.” But this begs the question; what is “serious loss or detriment?” This ambiguity is prevalent in the affordability literature. For example, Kroshl and Pandolfini (2000) note that

> No single formula precisely defines an affordable system. As a micro-concept, an affordable system is procured when needed within a budget, operated at a desired performance level, and maintained and supported within an allocated life-cycle budget. As a macro-concept, affordable systems are constrained by top-

> line budgets, require timing for competing uses of resources, and must contend with the dimension of inflexibility in near-term budgets, although long-term considerations may make many programs justifiable.

Redman and Straton (2001) define affordability as

> that characteristic of a product or service that enables consumers to: (1) *procure* it when they need it; (2) *use it to meet their performance* requirements at a level of quality that they demand; (3) *use it whenever they need it* over the expected life span of the product or service; and (4) *procure it for a reasonable cost* that falls within their budget for all needed products or services.

With regard to defense programs, the relatively recent emphasis on affordability[1] is in marked contrast to the Department of Defense's behavior during the Cold War. Then, the emphasis was on effectiveness, and cost, if considered at all, was just another variable. The end of the Cold War brought a defense drawdown and accompanying budget cuts, causing an increased emphasis on cost in resource allocation decisions. This emphasis was formalized in a July 19, 1995, memo titled "Policy on Cost-Performance Trade-Off" signed by the then Under Secretary of the U.S. Department of Defense for Acquisition and Technology. The memo introduced the cost-as-an-independent-variable (CAIV) initiative. CAIV mandated that decisions be made considering both total life-cycle costs (TLCC) and effectiveness as the decision variables. While CAIV made TLCC visible, it allowed for trade-offs between effectiveness and TLCC. Thus, if decision-makers put enough weight on effectiveness, they could still approve systems that were not necessarily affordable, leading the DoD to revise the concept of affordability. The DoD defines affordability as "the degree to which the life-cycle cost of an acquisition program is in consonance with the long-range investment and force structure plans of the Department of Defense or individual DoD Components."

It is interesting that the U.S. Department of Defense has difficulty in identifying what is or is not affordable. In our private lives, we all know implicitly what affordability means. For example, in deciding whether to buy a new car, I have many options, including keeping my old car rather than buying a new one. To make my decision, I must decide how much I am willing to spend on a car as well as decide what attributes I desire in a car. One of the options I could consider might be a luxury car. In evaluating the affordability of a luxury car, I would determine whether it fit into my budget. At this point, things get a little complicated. What does fit in my budget mean? It might mean that I have already determined the maximum amount I am willing to spend on a car. In that case, the luxury car either costs no more than my pre-determined amount or it doesn't (i.e., is affordable or isn't). On the other hand, I might have in mind an amount I am willing to spend on a car, but I might also be willing to make trade-offs with other items in my budget if the alternative exceeds my pre-determined amount. For example, I might be willing to forego eating in restaurants, going to the movies, etc., in order to buy the luxury car. In that case, I would want the utility of a combination of goods that includes the luxury car to offer at least as much utility as any combination of goods that does not include the luxury car. Therefore, the luxury car is affordable if, after making trade-offs, it fits in my budget and produces at least as much total utility as I would have without it. This leads to a workable definition of affordability: A system is affordable if, after making any desired tradeoffs, it fits in the budget and offers at least as much utility as the current mix of systems.

[1] For another view of affordability and an excellent review of the affordability literature, see Melese (2010).

Our goal in this paper is to develop an operational definition of affordability that lends itself to quantification. We first lay out a model of choice that is the foundation of modern economic reasoning. We next apply this formulation to a defense budget decision. Finally, we construct a quantifiable, operational affordability measure.

The Model

Assume that the government defense agency currently produces or purchases a good q_1 at a unit cost of c_1 that provides social utility (i.e., contributes to the defense of the country). Suppose a new good, q_2, at a unit cost of c_2, is available, where q_2 can be independent of, a substitute for, or a complement to q_1. Assume also that the agency currently has a budget of B, which it uses to purchase or produce a quantity q_1^* of the good q_1. Assuming a social utility function $U = f(q_1, q_2)$, the agency faces three possible situations. The first is that q_2 is not affordable, in which case the agency will not produce or purchase it. In the second situation, q_2 is affordable, and social utility is maximized by partially substituting q_2 for q_1. In this case, both q_1 and q_2 will be produced or purchased. In the third case, q_2 dominates q_1 and social utility is maximized by discontinuing q_1 and producing or purchasing only q_2. We examine each case in more detail below.

Case 1: q_2 Is Not Affordable

If $c_2 > B$, q_2 is clearly not affordable unless funds are available to increase B. If the situation is as shown in Figure 1, however, the affordability of q_2 is not clear. In Figure 1, q_1^* is the quantity of q_1 given a budget level of B and a price of c_1 with q_2 not included (i.e., it is the status quo). Including q_2 results in the budget line and social indifference curves shown. $U = f(q_1^*, 0)$ is the indifference curve representing the combinations of q_1 and q_2 that produce as much utility as q_1^*. Note that there are combinations of q_1 and q_2 that satisfy the budget constraint. Many of the definitions of affordability noted above define affordability by whether it fits within the budget constraint. Under these definitions, q_2 is affordable. By our definition of affordability, however, q_2 is not affordable because, whereas q_2 fits within the budget constraint, adding q_2 does not achieve a utility level at least as high as q_1^*. There is no combination of q_1 and q_2 that both satisfies the budget constraint and produces as much utility as q_1^*. With a budget level of B, adding q_2 results in lower utility; therefore, q_2 does not meet our definition of affordability, and q_2 is not produced. This explains a lot of seemingly paradoxical behavior. For example, in the debate over health care some note that there are families and individuals who seem to have enough income to purchase health insurance yet do not. The reason is that their expected level of utility with health insurance in their mix of consumption goods is lower that the mix without it given their income.

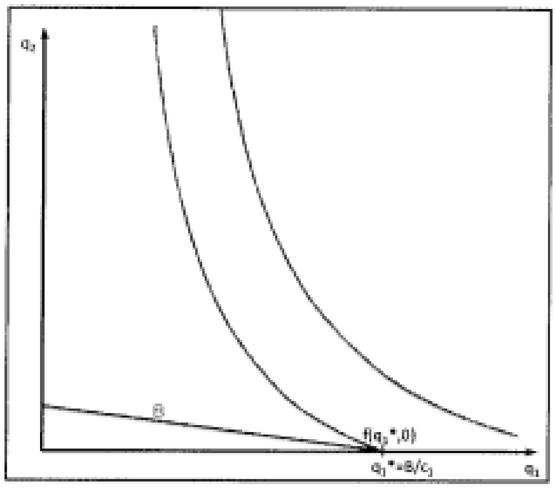

Figure 1. Case 1

Case 2: q_2 Is Affordable, Leading to the Production or Purchase of Both q_1 and q_2

This case is shown in Figure 2. As before, $U = f(q_1^*, 0)$ is the indifference curve showing the combinations of q_1 and q_2 that produce as much utility as q_1^*. In this case, however, there are combinations of q_1 and q_2 within the budget constraint that produce as much or more social utility as does q_1^*. Point A is the combination of q_1 and q_2 that produces maximum utility for a budget level of B; however, any combination of q_1 and q_2 in the area between the indifference curve $U = f(q_1^*, 0)$ and the budget line B (shaded area) produces more utility and costs as much or less than q_1^*. Thus, q_2 is affordable.

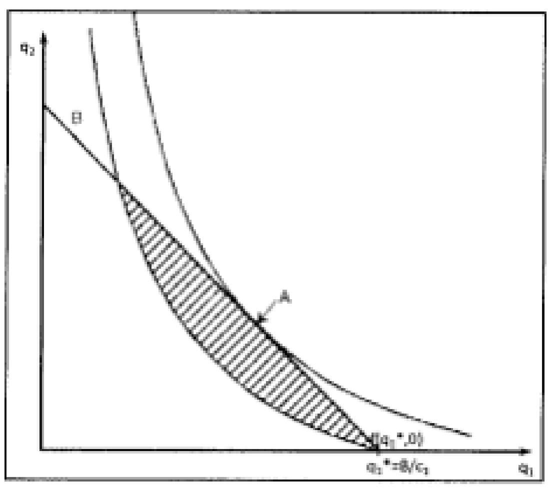

Figure 2. Case 2

Case 3: q_2 Dominates q_1

Given the contours of the indifference curves and the slope of the budget line in Figure 3, q_2^* satisfies the budget constraint while providing more utility than q_1^* or any combination of q_1 and q_2 that satisfies the budget constraint. Indeed, any quantity of q_2 between q_2' and q_2^* costs less and provides more utility than does q_1^*. In this case, only q_2 is produced.

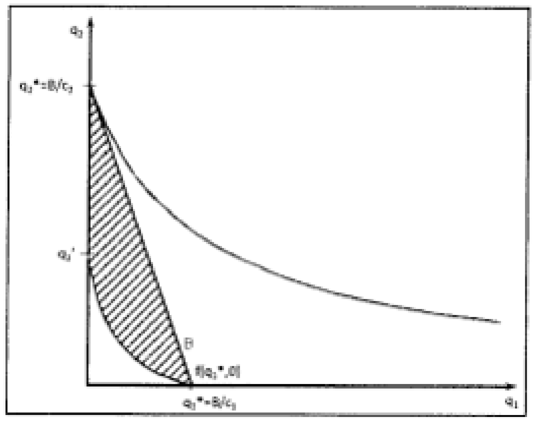

Figure 3. Case 3

In all three cases, the combinations of q_1 and q_2 that cost as much or less than the status quo lie in the triangle formed by the budget line and the q_1 and q_2 axes, that is, by the origin q_1^* and q_2^*. The combinations of q_1 and q_2 that meet our definition of affordability are contained in the area between the budget line and the indifference curve (the shaded area in Figures 2 and 3). This leads directly to a quantifiable measure of affordability: It is the ratio of the set of combinations of q_1 and q_2 that produce at least as much utility as q_1^* to the set of all combinations of q_1 and q_2 that cost as much or less than q_1^*. This ratio defines an affordability index a such that $0 \le a < 1$.

Extensions

More Than One Good in the Status Quo Budget

In this case, q_1 represents a vector of goods. Let q_i, $i = 1,...,n$ be the existing goods, with q_1^* representing the optimal mix of the current goods. Introducing q_2 into the mix leads to the same three cases described above except that the tradeoffs are now among multiple goods.

Affordability Over Time

Many affordability decisions involve long-lived assets and therefore affordability must be assessed over multiple time periods. Affordability over time is much more complicated than affordability over a single time period. In this situation, every time period must be assessed for the existence of the three cases above. By our definition of affordability, if in any one-time period the first case holds, q_2 is not affordable. However, if it is possible to alter the budget in a particular period where q_2 is not affordable so that it becomes affordable without making it unaffordable in another period, then it meets our criteria for affordability.

Illustrative Example

Let the measure of effectiveness for each alternative system be described by an exponential function with two nonnegative parameters a_i and c_i:

$$v_i(q_i) = 1 - e^{-g_i(q_i)} \tag{1}$$

where

$$g_i(q_i) = a_i q_i^{b_i}.$$

The parameter a_i determines the rate at which v_i increases with $q_i^{b_i}$. The parameter b_i affects the shape of v_i in that $b_i > 1.0$ produces S-shaped curves while $b_i \le 1$ produces concave growth curves. This function is general enough to exhibit both increasing and decreasing marginal effectiveness.

Let the joint effectiveness of the two systems be described by

$$v(q_1, q_2) = 1 - e^{-f(q_1, q_2)}$$

where

$$f(q_1, q_2) = g_1(q_1) + g_2(q_2) + d \cdot g_1(q_1) \cdot g_2(q_2). \tag{2}$$

The parameter d represents synergistic affects between q_1 and q_2. If $d > 0$ then q_1 and q_2 reinforce one another and produce a higher measure of effectiveness for the same (q_1, q_2) than when $d = 0$. This joint function exhibits two important traits. First, it exhibits eventually decreasing marginal effectiveness along any direction in the $(q_1, q_2)-$plane. Second, $v(q_i, q_j) \to v_i(q_i)$ as $q_j \to 0$ so that the joint measure of effectiveness reduces to the appropriate individual measure of effectiveness when one alternative is removed. These effectiveness measures are defined for all non-negative (q_1, q_2) but we will restrict our consideration to only integer values of q_1 and q_2.

Suppose the current system is such that $a_1 = 0.17$ and $b_1 = 0.6$ while the new, more effective, system is such that $a_2 = 0.21$ and $b_2 = 0.8$. Also assume $d = 0.3$. This produces a joint measure of effectiveness with the effectiveness contours presented in Figure 4.

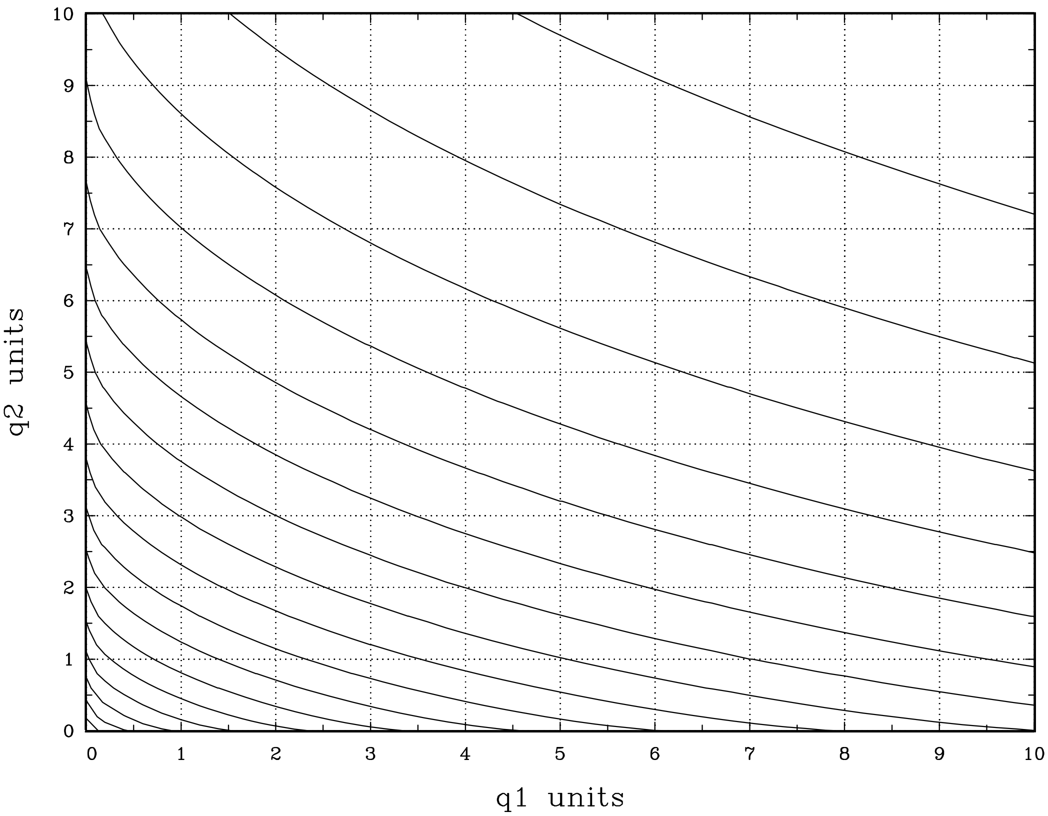

Figure 4. Contour Plot for $v(q_1, q_2)$ of Equation 2

Suppose the budget is $B = 10$ and assume that the current system inventory is $q_1 = 10$ so $v_1(10) = 0.492$. Is the new alternative affordable? To answer this we need to specify c_2. Suppose there are four cases to consider: $c_2 = 1, 2, 5$ and 10. Each produces a budget constraint line between $(q_1, q_2) = (10, 0)$ and $(q_1, q_2) = (0, q_2^{\max})$, where $q_2^{\max} = 10, 5, 4, 2$ or 1, respectively.

The new system is affordable if there are (q_1, q_2) combinations for which

$$v(q_1, q_2) \geq v(10, 0) = v_1(10) = 0.492 \quad (3)$$

and

$$c_1 q_1 + c_2 q_2 \leq 10. \quad (4)$$

Figure 5 presents the situation graphically. The region for which $v(q_1, q_2) \geq 0.492$ is depicted by the set of closely spaced contours (at intervals of 0.004). All combinations of (q_1, q_2) for which $v(q_1, q_2) < 0.492$ occupy the region with no contours. The four regions for which (q_1, q_2) satisfy Equation 4 are identified by their respective lines. The situation shows that q_2 is not affordable when $c_2 = 10$. There are no integer solutions other than $(q_1, q_2) = (10, 0)$, that satisfy Equation 3 and Equation 4. If $c_2 = 5$, we find q_2 is affordable at $(q_1, q_2) = (5,1)$. If $c_2 = 2$, we find 14 integer solutions, other than $(q_1, q_2) = (10,0)$, that are affordable. Finally, if $c_2 = 1$, there are 44 combinations, other than $(q_1, q_2) = (10,0)$, that are affordable. The answer to the question of whether the new system is affordable clearly depends on c_2. If $c_2 = 10$, then "no." But if $c_2 < 10$, then "yes."

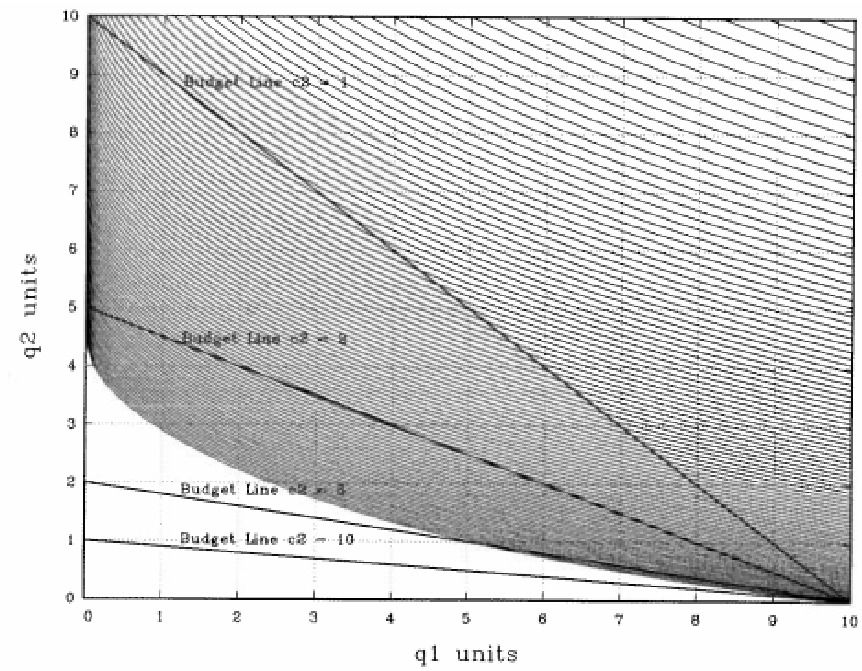

Figure 5. The Affordability of q_2 when c_2 = 10, 5, 2, and 1.0

If a system is deemed affordable, then the next question we ask is: How affordable is it? The answer is given by the affordability measure we developed in Section 2. The affordability measure is given by the area defined by Equation 3 and Equation 4 relative to the area defined in Equation 4 alone. The calculation of this area is an exercise in freshman calculus, but a useful approximation obtained by simple computation. All we need do is cover the area $\Omega = \{(q_1, q_2) \mid 0 \le q_1 \le 10; 0 \le q_2 \le 10\}$ with a grid of equally spaced points and count how many grid points lay within each area. If $N =$ the number of grid points satisfying Equation 3 and Equation 4, and $M =$ the number of grid points that lay on or below the respective budget line, then $A = N / M$ is an estimate of the measure of affordability. The finer the grid, the better the approximation. Table 1 illustrates this effect. $A^{(x)}$ denotes the value of A obtained using a grid of width x. The third column is the result of using an integer-based grid while the fourth and fifth columns present estimates for A using an increasingly finer grid.

Table 1. Affordability Measure

c_2	q_2	$A^{(1)}$	$A^{(0.1)}$	$A^{(0.01)}$
1.00	10	0.733	0.736	0.753
2.00	5	0.400	0.474	0.507
2.50	4	0.259	0.348	0.384
3.33	3	0.100	0.242	0.215
5.00	2	0.063	0.047	0.054
10.00	1	0.000	0.002	0.001

Note that N is an approximation to the area $\mathbf{N} = \{(q_1, q_2) \mid v(q_1, q_2) \geq v(0,10)$ and $B \geq c_1 q_1 + c_2 q_2\}$, while M is an approximation to the area $\mathbf{M} = \{(q_1, q_2) \mid B \geq c_1 q_1 + c_2 q_2\}$. Because $\mathbf{N} \subset \mathbf{M}$, it will always be the case that

$$0 \leq A \leq 1.$$

Affordability depends on more than just the alternative cost, c_2, and it is of value to use the model to study the effect of variations in other factors. For example, what is the change in the situation if the effectiveness of q_2 is further enhanced? Suppose the design of the alternative system can be improved so that $a_2 = 0.31$ and c_2 while all other parameters remain the same. This situation yields the measures of affordability in Table 2. We now find that even at $c_2 = 5.0$ the new system has a modest measure of affordability.

Table 2. Affordability Measure (a_2 = 0.31)

c_2	q_2	$A^{(0.01)}$
1.00	10	0.848
2.00	5	0.696
2.50	4	0.620
3.33	3	0.494
5.00	2	0.260
10.00	1	0.020

Of course, there are many other possibilities to consider. Not only is it of interest to understand the affect of the variation in a single variable, but also the affect of a combination of variables varying simultaneously. In the end we require a complete sensitivity analysis. Instead of pursuing these matters here, we prefer to consider the incorporation of uncertainty. To a certain extent a study of uncertainty and its effects on affordability is quite similar to a sensitivity analysis, but more focused.

Affordability-Effectiveness Analysis

Our paired-comparison development of affordability extends easily to the situation where we have multiple competing alternatives. Let there be a set of K candidate alternatives, each described by their overall effectiveness and discounted life-cycle cost: $\{v_k, c_k; k = 1, K\}$. The relative cost-effectiveness of the members is assessed in the usual way by viewing a scatter diagram plot of the members of this set in cost-effectiveness space (i.e., a plot of v_k versus c_k). We wish to replace this plot with a scatter diagram plot in affordability-effectiveness space; that is, a plot of v_k versus A_k. This is achieved by repeating the paired-comparison analysis process for each of the candidate systems in order to obtain their description in terms of the ordered pairs $\{v_k, A_k; k = 1, K\}$. We illustrate this using the data of the previous section.

Let the original system and the three candidate systems be described as before by Equation 1 and Equation 2 with parameters as given in Table 3.

Table 3. Multiple Candidate System Example

k	a_k	b_k	c_k
0	0.17	0.6	1.0
1	0.21	0.8	2.0
2	0.35	0.7	5.0
3	0.41	0.9	10.0

The evaluation of the three new systems gives $A_1 = 0.495, A_2 = 0.343, A_3 = 0.054$. The respective cost-effectiveness and affordability-effectiveness plots are presented in Figure 6.

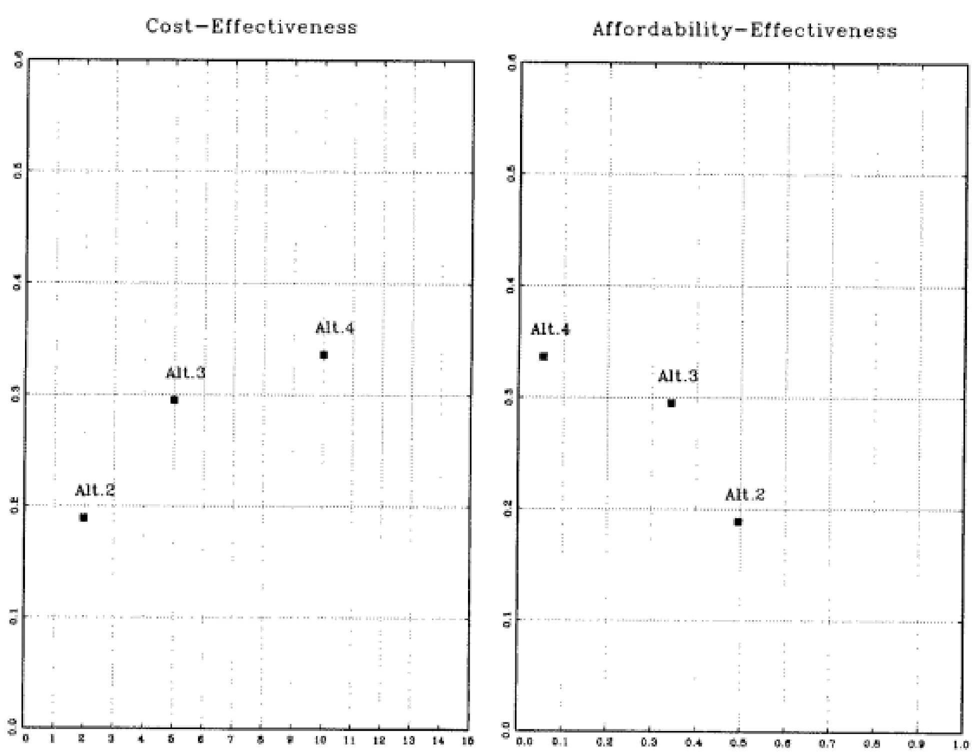

Figure 6. Cost-Effectiveness and Affordability-Effectiveness

Note how each may be viewed as the mirror image of the other. Both exhibit an efficient frontier, although with the opposite orientation with respect to the preferences of the horizontal axis. In the cost-effectiveness plot the preferred direction is upward and to the left while in the affordability-effectiveness plot the preferred direction is upward and to the right.

If the apparent mirror image of the two plots of Figure 6 is a pattern that always appears, then one could argue that the information produced by the affordability analysis offers nothing beyond the information contained in the cost-effectiveness analysis. The efficient set is the same in both plots and the trade-offs are mirror images of one another: Is the increase in effectiveness in choosing Alternative 3 over Alternative 2 worth the increase in cost? As opposed to: Is the increase in effectiveness in choosing Alternative 3 over Alternative 2 worth the loss in affordability? A small change in our example shows this not to be the case. Let the cost of Alternative 2 increase to $c_2 = 3.0$ and let that cost of Alternative 4 decrease to $c_4 = 9.0$. Application of our analysis to this new situation gives the results presented in Figure 7. Now we find a different efficient set in affordability-effectiveness space. In fact, Alternative 2 is now no longer efficient—it is dominated by Alternative 3. This is a significant alteration of the cost-effectiveness situation: the efficient set is now composed of only Alternatives 3 and 4. The decision to be made now concerns only two alternatives.

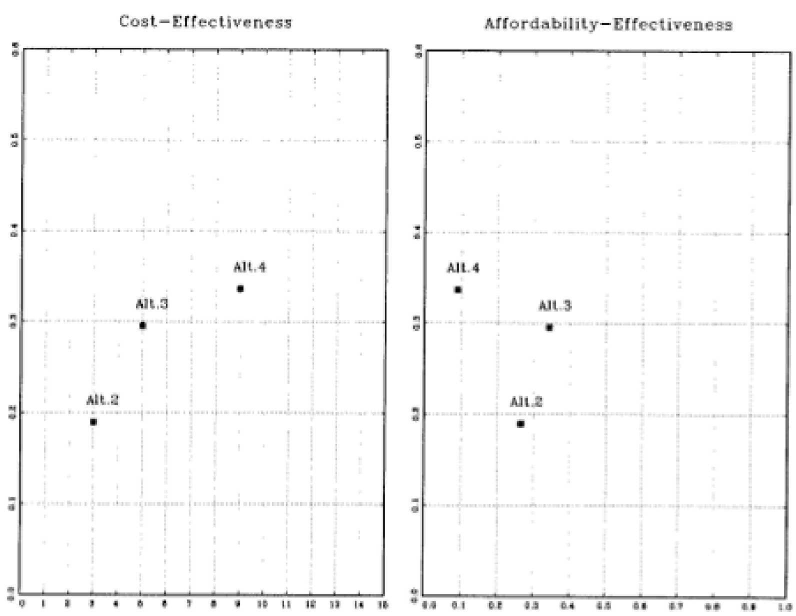

Figure 7. Cost-Effectiveness and Affordability-Effectiveness With c_2 = 3.0 and c_4 = 9.0

Uncertainty and Affordability Risk

Our affordability concept accommodates uncertainty in an obvious way. All we need to do is interpret the occasion when $(q_1, q_2) \in \mathbf{N}$ as an event and the metric $A = N / M$ as a random variable. Now the determination of an alternative's affordability is equivalent to calculating the $P\{\mathbf{N} \neq 0\} = 1 - P\{\mathbf{N} = 0\}$ or determining the $P(A > 0) = 1 - P(A = 0)$. Since affordability is a binary concept—either $(q_1, q_2) \in \mathrm{N}$ or not—an alternative is affordable if there exists only a single point for which $(q_1, q_2) \in \mathbf{N}$.

Of more use to the decision-maker is an assessment of the affordability measure. An alternative with a high measure of affordability implies that there are many combinations of (q_1, q_2) that will be preferred to the status quo. In the presence of uncertainty this can mean a higher likelihood for a satisfactory outcome—one in which the chosen (q_1, q_2) actually produces at least as high effectiveness as the status quo. Thus, it is of some interest to the decision-maker to ascertain $P(\alpha_L \leq A \leq \alpha_H)$ for various (α_L, α_H). This is equivalent to assessing the quantiles of A and this requires the distribution function of A. The assessment of affordability risk now takes explicit form.

- AFFORDABILITY RISK (Type 1). The likelihood that an alternative is unaffordable:

$$P(A = 0).$$

- AFFORDABILITY RISK (Type 2). The probability that the measure of affordability is less than some minimally acceptable level:

$$P(A < \alpha_{\min}).$$

The assessment of both types can be accomplished in many ways, but we find simulation modeling particularly attractive.

Simulation modeling makes good use of all available information concerning the uncertainties of the situation. It incorporates available theoretical results, subjectively assessed information, and assumptions the decision-maker is willing to make to fill in the gaps in required information. In our present context, there often are probability models representing estimations errors, particularly if life-cycle cost estimates rely on statistical techniques as regression in building cost estimating relations (CER). Moreover, the analyst often has knowledge of the measurement errors and imprecision in the evaluation of effectiveness.

An Illustration of Affordability Risk Assessment

We now illustrate the simulation modeling approach using our previous, deterministic example. The main issue of concern is computation of the probability distribution of A. This is all the information we need to assess any statistic relating to A, especially those we use to represent our two measures of risk. Simulation modeling provides only an approximation of the statistic of interest, but the accuracy of this approximation is limited only by the amount of time and computation we allocate to the task.

All parameters relating to the existing system are assumed known with certainty: $q_1 = 10, a_1 = 0.17, b_1 = 0.6, c_1 = 1$. The nominal values for the new system are as before: $a_2 = 0.21, b_2 = 0.8$ and $d = 0.3$. Cost is considered uncertain within the range: $1 \leq c_2 \leq 10$. Although the decision-maker is willing to believe the certainty attached to the parameters of the existing system, all parameters of the new system are viewed as only nominal.

We present six runs illustrating risk assessment scenarios under a variety of input specifications. Each illustrates the type of information the decision-maker may use: (1)

assumptions based on little or no prior information, (2) subjective assessment of related information, or (3) available hard data provided by the analyst (e.g., life-cycle cost estimation error and effectiveness estimation errors). The first three scenarios depict a situation where the decision-maker is willing to accept the effectiveness estimate for the new system (q_2) but not its cost estimate nor the value of the future budget. Run 1 assumes the decision-maker is willing to state a value for the minimum, most likely, and maximum value for c_2 and B. This type of prior information can be expressed as a triangular probability distribution or a Beta distribution parameterized to accept specification in three-parameter form (instead of the traditional two-parameter form). This type of Beta is referred to as a Program Evaluation and Review Technique (PERT) distribution. In Run 2 the decision-maker is willing to specify only a minimum and maximum for c_2 while believing that any value between these limits is equally likely. This information is represented by a uniform probability distribution. Run 3 extends this less informative prior to the budget as well. The last three runs illustrate the situation when the decision-maker no longer accepts the effectiveness estimate for the new system but is willing to employ the parameters as the most likely values in PERT distributions. Runs 4 and 5 illustrate pessimistic views of the new system effectiveness estimate. Run 6 illustrates the amount of improvement required in a_2, relative to Runs 4 and 5, to reduce the risks to acceptable levels (assuming a decision-maker who can tolerate a level as high as 0.05 or 5%). Each run employs 5,000 Monte Carlo trials with Roman hypercube sampling.

Table 4. Simulation Scenarios

RUN	a_2	b_2	c_2	d	B
1	0.21	0.80	PERT(2.0, 2.2, 5)	.3	PERT(9, 10, 10.05)
2	0.21	0.80	Uniform(2, 5)	.3	PERT(9, 10, 10.05)
3	0.21	0.80	Uniform(2, 5)	.3	Uniform(9, 10.05)
4	PERT(.17, .21, .22)	PERT(.65, .80, .85)	Uniform(2, 5)	.3	Uniform(9, 10.05)
5	PERT(.17, .21, .22)	PERT(.65, .80, .85)	Uniform(2, 5)	.3	Uniform(8, 10.05)
6	PERT(.20, .35, .40)	PERT(.65, .80, .85)	Uniform(2, 5)	.3	Uniform(8, 10.05)

The results are presented in Table 4 using four statistics. Columns 2 and 3 are the limits of the 95% confidence interval for A. These define the limits of the interval on the real line within which we will experience the actual (realized) value of A. The fourth column gives the estimate of Type 1 Risk (i.e., the likelihood the new system will be unaffordable). The last column presents the estimate of Type 2 Risk (i.e., the likelihood that actual A will be less than what is minimally acceptable—be specified as 0.1). The relative frequency distributions of A are presented in Figures 8–13.

Table 5. Affordability Statistics

RUN	$\alpha_{(.025)}$	$\alpha_{(.975)}$	$P(A=0)$	$P(A \leq 0.1)$
1	0.125	0.475	0.000	0.010
2	0.026	0.475	0.000	0.298
3	0.000	0.467	0.023	0.343
4	0.000	0.452	0.030	0.355
5	0.000	0.425	0.211	0.502
6	0.061	0.693	0.003	0.041

Comparing the results of Runs 1–2 demonstrates an important insight for decision-makers: the amount of probabilistic information provided affects the assessed risk. Both the PERT and uniform distribution have the same range of values but the PERT distribution provides more information: the most likely value, as well as the upper and lower bounds. As a consequence, the PERT distribution decreases the likelihood of values at, or near, the extremes of the distribution while placing more likelihood on values nearer the most likely value. This manifests itself in less assessed risk: a more narrow 95% confidence interval for A and a very small value for $P(A \leq 0.1)$. Using the uniform distribution for c_2 represents a reduction in information and leads to more assessed risk: a wider confidence interval for A shifted towards zero and higher $P(A \leq 0.1)$. Run 3 represents a further reduction in information and increase in assessed risk: the 95% confidence interval for A now includes zero with $P(A=0)=0.029$ and $P(A \leq 0.1)=0.404$.

Runs 4–5 illustrate the situation where the decision-maker does not have complete confidence in the estimate of effectiveness for the new system. Actual a_2 may be as much as 19% below the nominal while only 5% above the nominal, but its most likely value at the nominal estimate. Likewise, b_2 may be as much as 19% below nominal or 6% above with a most likely value at the nominal estimate. Run 4 is to be compared with Run 3 to see the effect on risk when uncertainty in effectiveness is added to the analysis. Run 5 is to be compared with Run 4 to see the effect of an even more pessimistic budget environment.

Run 6 addresses risk from a different perspective. The question here is the amount of increased effectiveness that must be offered by the new system to lower the risk to an acceptable level. We illustrate using only a_2 to keep a narrow scope. We find the new system reduces risk significantly if $0.20 \leq a_2 \leq 0.40$ with most likely value 0.35. This produces $P(A=0) \leq .01$ whereas $P(A \leq 0.1)$ □ $0.05,$ presenting a considerably less risky situation than Runs 4 and 5. This result is only suggestive—a more detailed analysis also involving b_2 would be required to more completely answer the question.

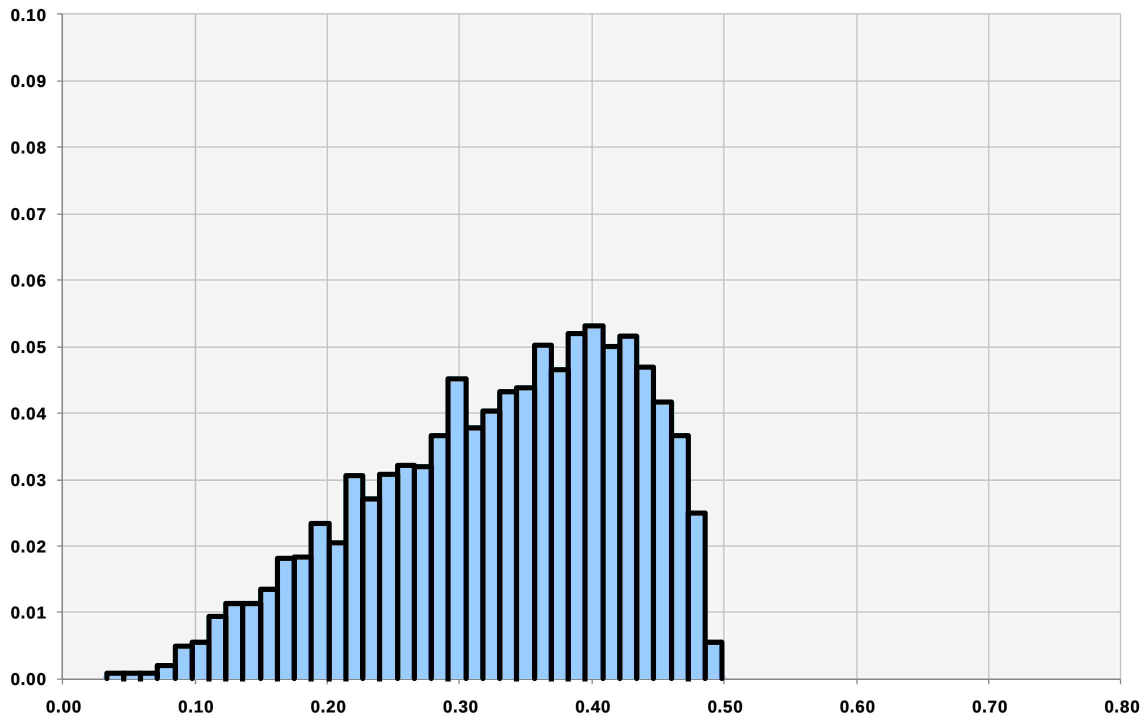

Figure 8. Run 1 Affordability Measure

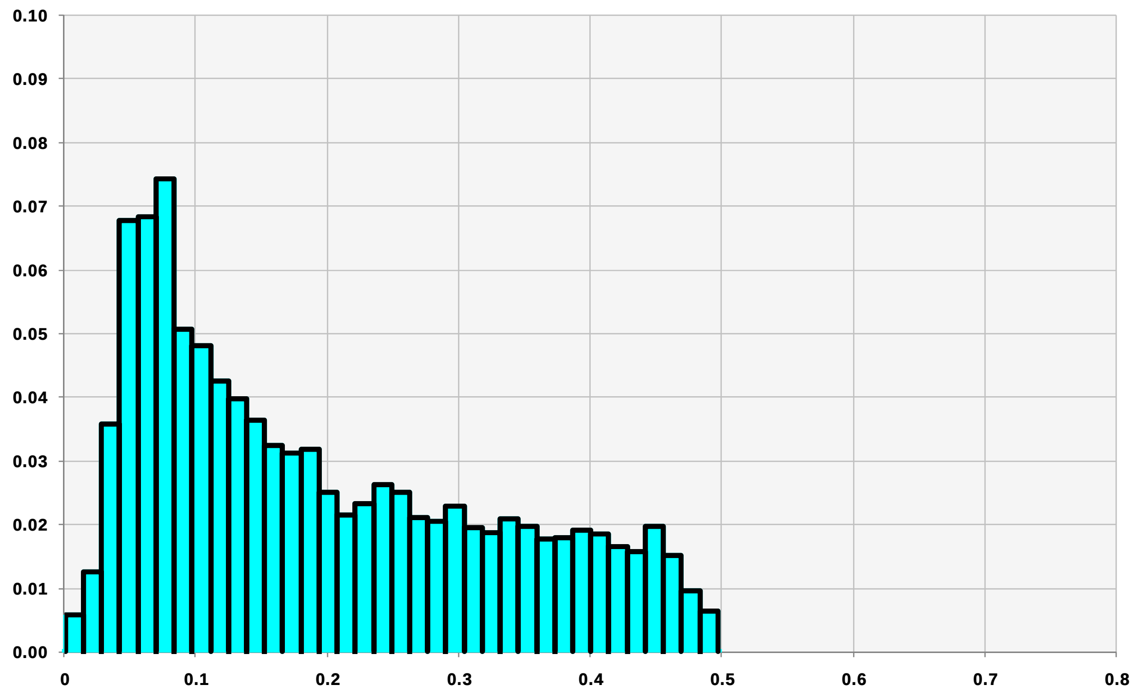

Figure 9. Run 2 Affordability Measure

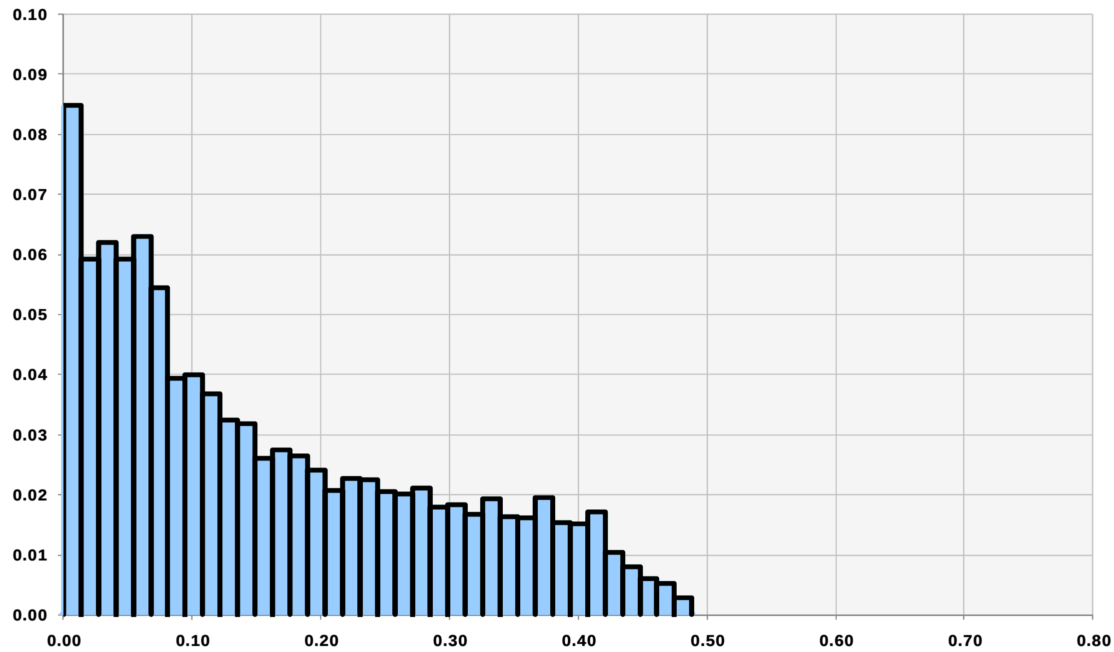

Figure 10. Run 3 Affordability Measure

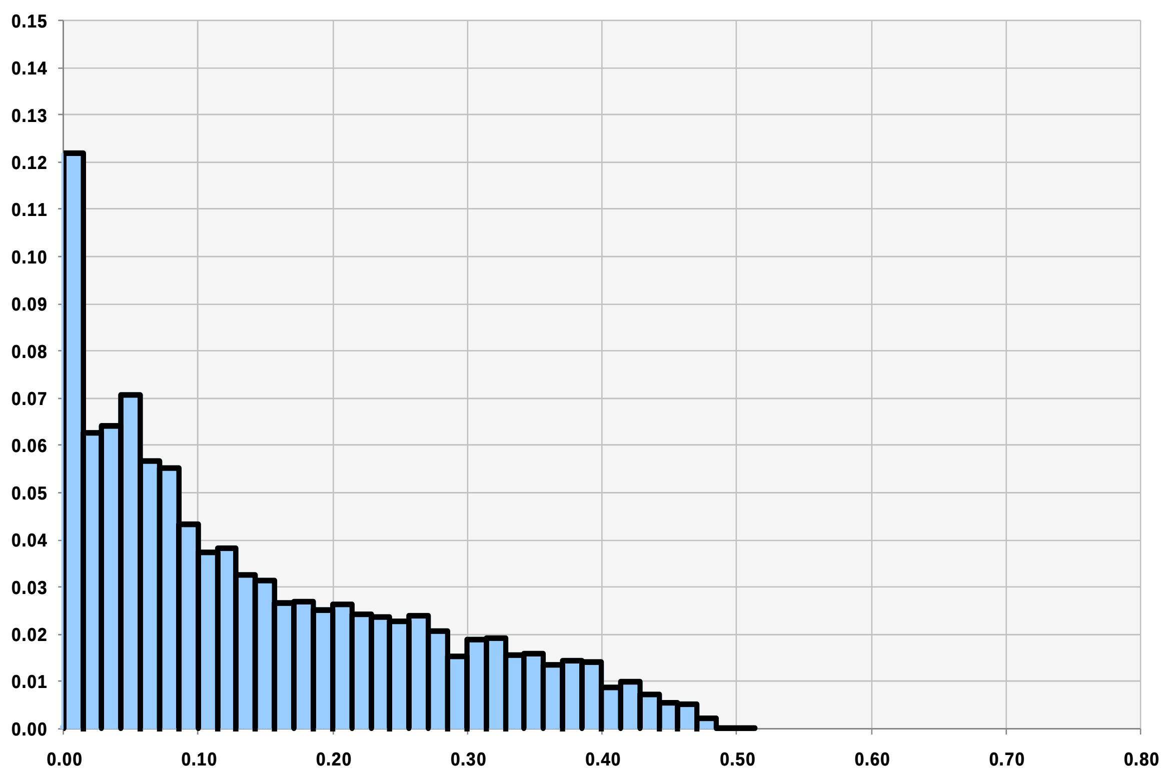

Figure 11. Run 4 Affordability Measure

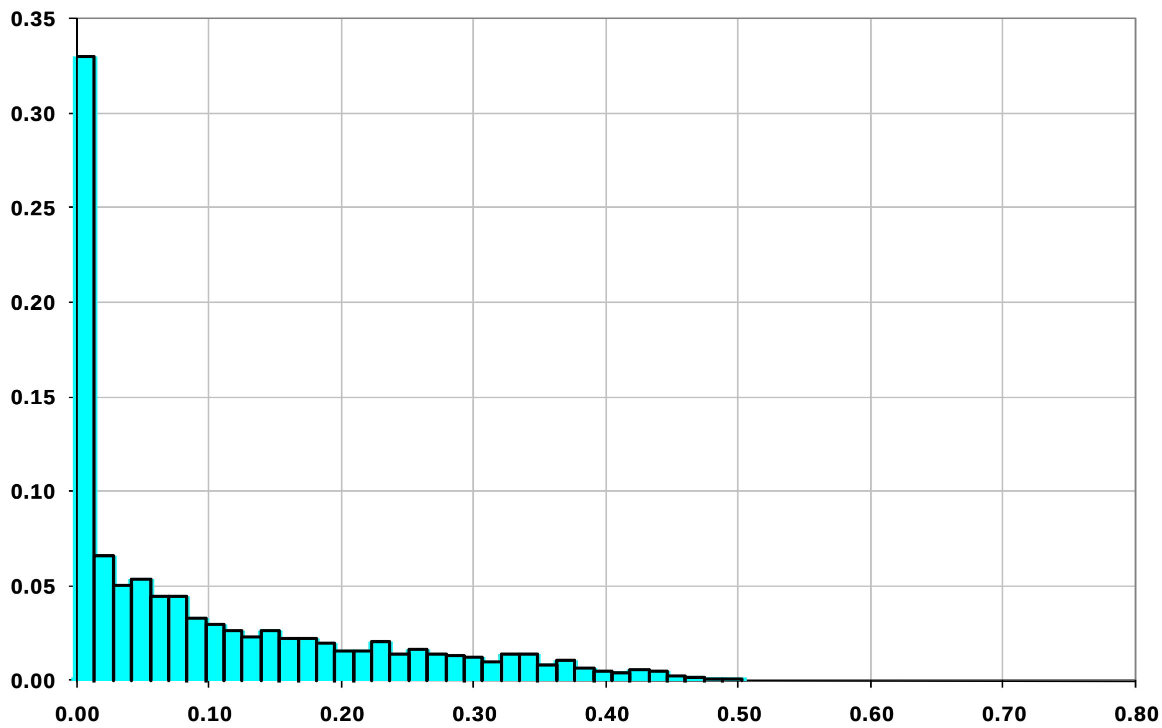

Figure 12. Run 5 Affordability Measure

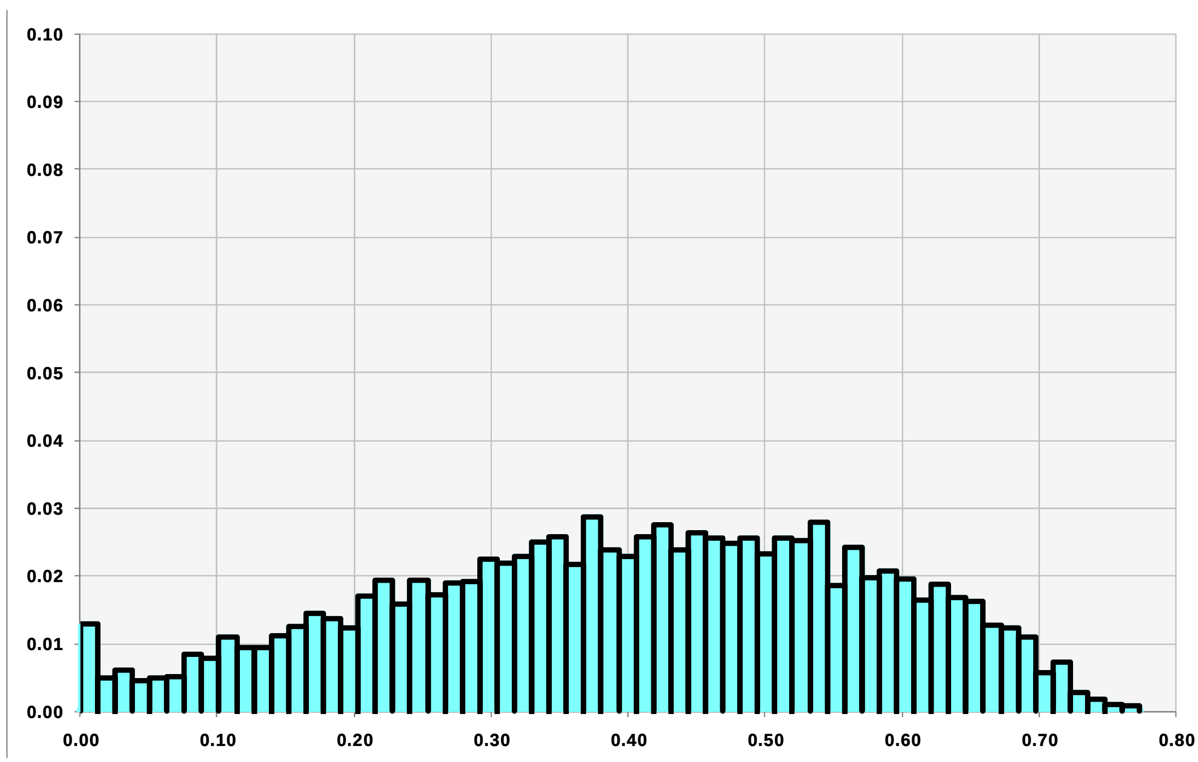

Figure 13. Run 6 Affordability Measure

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